Nonmonotonic Inheritance, Argumentation and Logic Programming

Phan Minh Dung and Tran Cao Son

Computer Science Program
School of Advanced Technology
Asian Institute of Technology
G.P.O Box 2754, Bangkok 10501, Thailand
Email: dung@cs.ait.ac.th, tson@emailhost.ait.ac.th

Abstract. We study the conceptual relationship between the semantics of nonmonotonic inheritance reasoning and argumentation. We show that the credulous semantics of nonmonotonic inheritance network can be captured by the stable semantics of argumentation. We present a transformation of nonmonotonic inheritance networks into equivalent extended logic programs.

1 Introduction

Argument-based approaches to nonmonotonic reasoning have been intensively studied and became prominent in AI and Logic Programming [6, 21, 24, 1, 20] just recently. But reasoning based on arguments represented as paths, has been studied in nonmonotonic inheritance reasoning, a specific field of nonmonotonic reasoning, from the very first day [30] and then in [13, 15, 27, 28, 29, 26, 25, 8]. Path-based reasoning approaches to nonmonotonic inheritance networks are widely accepted because they are intuitive and easy to implement.

The interesting and surprising problem here is that the argument-based semantics of nonmonotonic inheritance network [13, 15, 27, 28, 29, 26, 25, 8] and the general argumentation reasoning [6, 21, 24, 1, 20] seem to have conceptually little in common. Touretzky et al. went so far to claim that one of the fundamental principle of argumentation - *the use of reinstater* - can not be applied in nonmonotonic inheritance reasoning [29].

The relation between nonmonotonic inheritance reasoning and more general frameworks to nonmonotonic reasoning like default logic, autoepistemic logic, logic programming has been intensively studied in [3, 4, 14, 11, 7, 23, 16]. The basic idea of these works is to find a way to translate a nonmonotonic inheritance network into a "equivalent" theory of the respected nonmonotonic logic. But all of these transformations do not preserve the original semantics of nonmonotonic inheritance networks. Hence, conceptually, the relationship between the natural path-based semantics of nonmonotonic inheritance networks and other more general nonmonotonic logics such as default logic, autoepistemic logic, etc. remains unclear. The goal of this paper is to address this problem. We do that by studying the relationship between the argumentation framework given in [1] and inheritance networks.

We show that each acyclic, consistent nonmonotonic inheritance network Γ can be viewed as an argumentation framework AF_Γ so that the credulous semantics of Γ "coincides" with the stable semantics of AF_Γ. Further, we prove that the grounded semantics of AF_Γ is contained in the skeptical semantics of Γ [13, 26]. Thus, we can say that grounded semantics provides the baseline of skepticism in inheritance reasoning.

We present a transformation of consistent nonmonotonic networks into extended logic programs and show that the credulous semantics of the former coincides with the answer set semantics [10] of the latter. To our knowledge, this is the first transformation of nonmonotonic inheritance network into other more general nonmonotonic logics preserving the semantics of nonmontonic inheritance networks.

2 Preliminaries

2.1 Inheritance network

A defeasible inheritance network Γ is defined here as a finite collection of positive and negative direct links between nodes. If x, y are nodes then $x \rightarrow y$ (resp. $x \not\rightarrow y$) represents a positive (resp. negative) *direct link* from x to y. A network Γ is *consistent* if there exist no two nodes x, y such that both $x \rightarrow y$ and $x \not\rightarrow y$ belong to Γ. A *positive path* from x_1 to x_n through x_2, ..., x_{n-1}, denoted by $\pi(x_1, \sigma, x_n)$ or $\pi(x_1, x_2, ..., x_{n-1}, x_n)$, is a sequence of direct links $x_1 \rightarrow x_2$, $x_2 \rightarrow x_3$, ... , $x_{n-1} \rightarrow x_n$. Similarly, a *negative path* from x_1 to x_n through x_2, ..., x_{n-1}, denoted by $\bar{\pi}(x_1, \sigma, x_n)$ or $\bar{\pi}(x_1, x_2, ..., x_{n-1}, x_n)$, is a sequence of direct links $x_1 \rightarrow x_2$, $x_2 \rightarrow x_3$, ... , $x_{n-1} \not\rightarrow x_n$. A generalized path is a sequence of direct links (x_1, x_2), (x_2, x_3), ... , (x_{n-1}, x_n), where (x_i, x_{i+1}) denotes a positive or negative direct link. Γ is *acyclic* if there is no generalized path (x_1, x_2), (x_2, x_3), ... , (x_{n-1}, x_n) with $x_1 = x_n$. The degree of the path $\alpha = \pi(x, \sigma, y)$ (resp. $\alpha = \bar{\pi}(x, \sigma, y)$), denoted by $deg_\Gamma(\alpha)$, is defined as the length (number of edges) of the longest generalized path from x to y. Furthermore, we also use the notation $\pi(x_1, \sigma, x_{n-1}) \rightarrow x_n$ (resp. $\pi(x_1, \sigma, x_{n-1}) \not\rightarrow x_n$) to denote the path $\pi(x_1, x_2, ..., x_{n-1}, x_n)$ (resp. $\bar{\pi}(x_1, x_2, ..., x_{n-1}, x_n)$).

From now on we will use Γ to denote an arbitrary but fixed network and Φ to denote a set of paths in Γ if no confusion is possible. The notion of inheritability presented here relies on three concepts: constructibility, conflict, and preemption. Their definitions are taken from [15, 29, 13, 25, 26].

Definition 1. A positive path $\pi(x, \sigma, u) \rightarrow y$ is *constructible* in Φ iff $\pi(x, \sigma, u) \in \Phi$ and $u \rightarrow y \in \Gamma$. A negative path $\pi(x, \sigma, u) \not\rightarrow y$ is constructible in Φ iff $\pi(x, \sigma, u) \in \Phi$ and $u \not\rightarrow y \in \Gamma$.

Definition 2. $\pi(x, \sigma, y)$ *conflicts* with any path of the form $\bar{\pi}(x, \tau, y)$ and vice versa. A path σ is conflicted in Φ iff Φ contains a path that conflicts with σ.

Different ways have been proposed to define defeasible preemption in Φ [7, 15, 13, 28, 26, 25]. Here, we follow the *off-path* preemption given in [13].

Definition 3. A positive path $\pi(x, \sigma, u) \rightarrow y$ (see figure 1) is *preempted* in Φ iff there is a node v such that (i) $v \not\rightarrow y \in \Gamma$ and (ii) either v=x or there is a path of the form $\pi(x, \tau_1, v, \tau_2, u) \in \Phi$. A negative path $\pi(x, \sigma, u) \not\rightarrow y$ is *preempted* in Φ iff there is a node v such that (i) either v=x or (ii) there is a path of the form $\pi(x, \tau_1, v, \tau_2, u) \in \Phi$ and $v \rightarrow y \in \Gamma$.

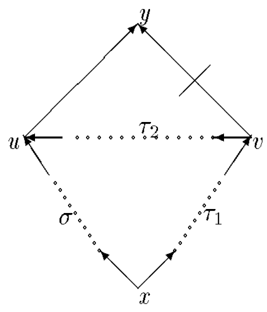

Fig. 1. $\pi(x, \sigma, u) \rightarrow y$ is preempted

The credulous semantics of an inheritance network is given in the following definition.

Definition 4. [13] σ is *defeasibly inheritable* in Φ, written as $\Phi \mid\!\sim \sigma$, iff
either σ is a direct link
or σ is a compound path, $\sigma{=}\pi(x, \tau, y)$ (likewise for negative path) such that
(i) σ is constructible in Φ, and
(ii) σ is not conflicted in Φ, and
(iii) σ is not preempted in Φ.

Definition 5. A set Φ of paths is a *credulous extension* of the net Γ iff $\Phi = \{ \sigma\colon \Phi \mid\!\sim \sigma \}$.

The skeptical semantics for inheritance network is defined by the notion of ideally skeptical extension and is defined as follows.

Definition 6. [26, 25] The intersection of all credulous extensions of Γ is called the *ideally skeptical extension* of Γ.

2.2 Argumentation framework

In the following section, the basics of the abstract theory of argumentation framework of Dung [1] is recalled.

Definition 7. An *argumentation framework* is a pair $AF{=}{<}AR, attacks{>}$, where AR is a set of arguments, and $attacks \in AR \times AR$.

If $(A, B) \in attacks$ we say A attacks B or B is attacked by A. A set of arguments S is said to be attacked by an argument A if there is $B \in S$ such that $(A, B) \in attacks$. Similarly, we say S attacks A if there is $B \in S$ such that $(B, A) \in attacks$.

Definition 8. A set of arguments S is said to be *conflict-free* if there exist no two arguments A,B in S such that $(A, B) \in attacks$.

The stable semantics of AF is defined as follows.

Definition 9. A conflict-free set of arguments S is called a *stable extension* of AF if S attacks every argument which does not belong to S.

It is easy to see that the following proposition holds.

Proposition. *S is stable iff S=$\{A \mid A$ is not attacked by $S\}$.*

The stable semantics of argumentation framework captures the semantics of many other mainstream approaches to nonmonotonic reasoning such as extension of Reiter's Default Logic [22], stable expansion of Autoepicstemic Logic [17], and stable model of Logic Programming [9]. We will see in section 3 that the credulous semantics of an inheritance network coincides with the stable semantics of a corresponding argumentation framework.

Often, a more restricted form of skeptical semantics is advocated in many approaches to nonmonotonic reasoning [21, 5]. This form of skeptical semantics is defined in the argumentation framework by the notion of grounded extension defined as the least fixpoint of the following operator.

$F_{AF} : 2^{AR} \rightarrow 2^{AR}$, where

$F_{AF}(\mathrm{S}) = \{\mathrm{A} \mid \forall\mathrm{B}$, if B attacks A, then $\exists\mathrm{C} \in S$ such that C attacks B$\}$.

The idea behind this operator will become clear in the next definition.

Definition 10. An argument A is *defendable* wrt S iff for every argument B, if B attacks A, then there is an argument C in S such that C attacks B.

So, we can redefine F_{AF} by $F_{AF}(S) = \{\mathrm{A} \mid \mathrm{A}$ is defendable wrt S$\}$.

The grounded extension of an argumentation framework is defined next.

Definition 11. The *grounded extension* of an argumentation framework AF denoted by GE_{AF} is the least fixpoint of F_{AF}.

It has been pointed out in [1] that both the semantics, Pollock's Inductive Defeasible Logic [21], and well-founded semantics of Logic Programming [5] are captured by the grounded extension of argumentation framework.

The maximal fixpoint of F_{AF} are called the preferred extension of AF. In general, stable extension are preferred extension but not vice versa. But as we will see later, for any argumentation framework corresponding to inheritance networks, stable semantics and preferred semantics coincide. So it is enough for us to work only with stable semantic.

3 Inheritance Networks as Argumentation Frameworks

Our goal is to clarify the conceptual relationship between the semantics of nonmonotonic inheritance networks and the semantics of argumentation frameworks. This will also help to illuminate the conceptual relationship between the semantics of nonmonotonic inheritance networks and other general approaches to nonmonotonic reasoning due to a result of Dung [1] showing that many general approaches to nonmonotonic reasoning [22, 17, 5] can be seen as special cases of argumentation frameworks.

We will show that every nonmonotonic inheritance network Γ can be considered as an argumentation framework $AF_\Gamma{=}{<}AR_\Gamma, attacks_\Gamma{>}$ such that the credulous semantics of Γ coincides with the stable semantics of $AF_\Gamma{=}{<}AR_\Gamma, attacks_\Gamma{>}$ in the sense that every credulous extension of Γ is a stable extension of AF_Γ and vice versa.

A path $\alpha = \pi(x, \sigma, u)$ is called a *prefix* of path $\beta = \pi(x, \sigma, u, \tau, v)$ in Γ. The set of all prefixes of β is denoted by $pre(\beta)$. α is a *proper prefix* of β iff $\alpha \in pre(\beta)$ and $\alpha \neq \beta$.

First, it is intuitive to view any path of Γ as an argument. So, we have $AR_\Gamma = \{\sigma | \sigma \text{ is a path in } \Gamma\}$.

As next we define the attacks relation of AF_Γ. The underlying principle in defining the attacks relation is that more specific information overrides less specific one. For two conflicted paths, σ and τ, there are following cases:

(i) σ is a direct link. It is clear that we should let σ attack τ, but not vice versa.

(ii) σ and τ are compound paths. In this case, neither σ nor τ are more specific than the other path. Thus, we have: σ attacks τ and vice versa.

Further, it should be also clear that if σ attacks a prefix α of τ then σ attacks τ.

We now consider another kind of attack.

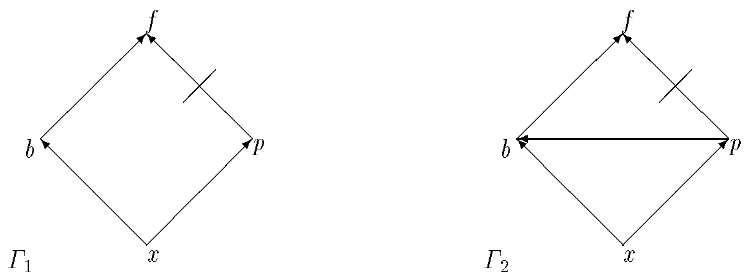

Fig. 2. Motivation of Attack Definition

Example 1. Consider the inheritance network Γ_1 in figure 2. The paths $\sigma = \pi(x,b,f)$ and $\tau = \bar{\pi}(x,p,f)$ are in conflict. Thus, $\pi(x,b,f)$ attacks $\bar{\pi}(x,p,f)$ and vice versa as in case (ii). So, AF_{Γ_1}=<$AR_{\Gamma_1}, attacks_{\Gamma_1}$> with $attacks_{\Gamma_1} = \{(\sigma,\tau),(\tau,\sigma)\}$. Hence AF_{Γ_1} has two stable extensions corresponding to two credulous extensions of $\Gamma_1 : E_1 = \Gamma \cup \{\sigma\}$ and $E_2 = \Gamma \cup \{\tau\}$.

The network Γ_2 in figure 2 is received from the network Γ_1 by adding the positive link $p \rightarrow b$. This is the well- known Penguin-Bird-Fly example. It is clear that in AF_{Γ_2}=<$AR_{\Gamma_2}, attacks_{\Gamma_2}$>, σ attacks τ and τ attacks σ as in case of Γ_1. Adding the link $p \rightarrow b$ into Γ_1 makes the argument τ more specific than the argument σ. Thus, due to the principle that more specific information can override less specific one, we can say that adding $p \rightarrow b$ to Γ_1 creates a attack of new kind against $\sigma = \pi(x,b,f)$. We can represent this by viewing the argument $\alpha = \pi(x,p,b)$ in the presence of the link $p \not\rightarrow f$ as an attack against the path $\sigma = \pi(x,b,f)$.

These motivations lead to the following definition of attacks.

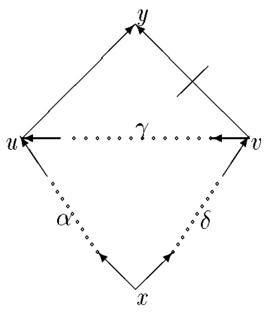

Fig. 3. $\pi(x,\alpha,u) \rightarrow y$ is attacked by $\pi(x,\delta,v,\gamma,u)$ in presence of $v \not\rightarrow y$

Definition 12. A path σ attacks path τ iff

(a) σ is a direct link x$\rightarrow$ y (resp. $x \not\rightarrow y$) and $\tau = \bar{\pi}(x,\alpha,y)$ (resp. $\tau = \pi(x,\alpha,y)$) or

(b) σ is in conflict with some compound path $\delta \in pre(\tau)$ or

(c) σ,τ are compound paths where there exists a prefix $\beta = \pi(x,\alpha,u) \rightarrow y$ of τ (resp. $\pi(x,\alpha,u) \not\rightarrow y$) and $\sigma = \pi(x,\delta,v,\gamma,u)$ with $v \not\rightarrow y \in \Gamma$ (resp. $v \rightarrow y \in \Gamma$) (see figure 3).

Remark. From now on we will refer to three types of attacks (a), (b), and (c) defined above as attack by direct link, by conflict, and by preemption, respectively.

So, in our point of view there are two kinds of attacks between two paths σ and τ, symmetric (τ attacks σ and σ attacks τ) and asymmetric (τ attacks σ but not vice versa). Symmetric attacks are equivalent to conflictor in [29] while asymmetric attacks have some similarity to preemptor in [29] but not identical.

Example 2. **(Continuation of example 1)**
For Γ_2 in figure 2 we have AF_{Γ_2}=<$AR_{\Gamma_2}, attacks_{\Gamma_2}$> with

$AR_{\Gamma_2} = \Gamma_2 \cup \{\sigma, \tau, \alpha, \beta\, \delta\}$ and

$attacks_{\Gamma_2} = \{(p \not\rightarrow f, \delta)\} \cup \{(\sigma,\tau),(\tau,\sigma),(\tau,\beta),(\beta,\tau)\} \cup \{(\alpha,\sigma),(\alpha,\beta)\}$ with $\sigma = \pi(x,b,f)$, $\tau = \bar{\pi}(x,p,f)$, $\alpha = \pi(x,p,b)$, $\beta = \pi(x,p,b,f)$, and $\delta = \pi(p,b,f)$. Here, $\{(p \not\rightarrow f, \delta)\}$ is the set of attacks by direct link, $\{(\sigma,\tau),(\tau,\sigma),(\tau,\beta),(\beta,\tau)\}$ is the set of attacks by conflict, and $\{(\alpha,\sigma),(\alpha,\beta)\}$ is the set of attacks by preemption. □

We now prove the coincidence between the credulous extension of Γ and the stable extension of AF_Γ.

Theorem 13. *Let E be a set of paths in Γ. Then, E is a stable extension of AF_Γ iff E is a credulous extension of Γ.*

Proof. See Appendix. □

We give now the definition of the grounded skeptical semantics for an inheritance network.

Definition 14. The grounded skeptical semantics of the inheritance network Γ is defined as the grounded extension GE_{AF_Γ} of the corresponding argumentation framework AF_Γ of Γ.

Since GE_{AF_Γ} is contained in every stable extension of AF_Γ we have the following theorem.

Theorem 15. *The grounded extension GE_{AF_Γ} of AF_Γ is a subset of the ideally skeptical extension of Γ.*

Proof. Since GE_{AF_Γ} is the smallest complete extension of AF_Γ and the complete extensions form a complete semilattice wrt set conclusions [1] we have that GE_{AF_Γ} is contained in every stable extension of AF_Γ. Thus, GE_{AF_Γ} is contained in their intersection which is the ideally skeptical extension. □

4 Transforming Inheritance Network into Logic Program

The coincidence between the credulous semantics of an inheritance network Γ and the stable semantics of the corresponding argumentation framework AF_Γ together with the results in [1] stating that argumentation frameworks in principle can be represented as logic programs, points out that an inheritance network Γ can be transformed into an equivalent logic program P_Γ. Thus, proof procedures based on negation-as-failure can be applied to P_Γ to compute the credulous semantics of Γ.

In this section we transform an inheritance network Γ into an extended logic program P_Γ and show that the credulous semantics of Γ coincides with the answer set semantics of P_Γ[1].

In following we assume that the readers are family with the answer set semantics of Logic Programs [10].

The set of nodes of Γ is the union of two disjoint sets, the set of individuals of Γ, denotes by I_Γ, consists of all nodes x of Γ such that there exists no direct link $y \rightarrow x$ or $y \not\rightarrow x$ in Γ, and the set of predicate (or properties) nodes. For example, in the example 3 (Nixon-Diamond), a denotes the individual Nixon, and p, r and q denote the predicates Pacifist, Republican and Quaker, respectively. In following, a, b, c, ... will represent the individuals of Γ and p, q, r, ... are the predicate nodes if not otherwise specified.

We first assign an unique natural number $j \in N$ to each direct link $p \rightarrow q \in \Gamma$ (resp. $p \not\rightarrow q \in \Gamma$) of Γ, $p \notin I_\Gamma$, written as $p {\rightarrow}_j q$ (resp. $p {\not\rightarrow}_j q$), and introduce a new predicate ab_j representing the abnormal-literal at the edge j. The link $p {\rightarrow}_j q$ (or $p {\not\rightarrow}_j q$) is then referred simply as the link j. Based on the attack relationship of AF_Γ the inheritance network Γ can be transformed into an extended logic program P_Γ as follows.

As in case of attack by direct link, any direct link $a \rightarrow p$ (resp. $a \not\rightarrow p$) beginning from a fact node a can be transformed directly into a fact of P_Γ because of there is no arguments which attack $a \rightarrow p$ (resp. $a \not\rightarrow p$). Hence, we have:

(i) For each $a \in I_\Gamma$ if $a \rightarrow p$ (resp. $a \not\rightarrow p$) is in Γ then

$$p(a) \leftarrow \text{(resp. } \neg p(a) \leftarrow)$$

is a clause of P_Γ.

(ii) For $p \notin I_\Gamma$ and each direct link $p {\rightarrow}_j q \in \Gamma$, the two clauses

$$\begin{array}{ll} q(x) & \leftarrow p(x),\ not\ ab_j(x) \quad \text{and} \\ ab_j(x) & \leftarrow \neg q(x) \end{array}$$

belong to P_Γ.

Similarly, we have two clauses of P_Γ for a negative direct link $p {\not\rightarrow}_j q \in \Gamma$ as follows:

(iii) For $p \notin I_\Gamma$ and each direct link $p {\not\rightarrow}_j q \in \Gamma$

$$\begin{array}{ll} \neg q(x) & \leftarrow p(x),\ not\ ab_j(x) \quad \text{and} \\ ab_j(x) & \leftarrow q(x) \end{array}$$

are clauses of P_Γ.

[1] Grégoire [7] presented an algorithm for transformation of an inheritance network into a logic program but in our view this could hardly be considered as a transformation because according to the algorithm we first have to compute the extensions of the network and then define a logic program specifying this extension.

(iv) For each pair of direct links $p \rightarrow_j q$ and $r \not\rightarrow_k q$ in Γ

(a) If there exists a positive path from p to r over the links $j_1, ..., j_n$ then the clause

$$ab_k(x) \leftarrow p(x),\ \mathit{not}\ ab_{j_1}(x), \ldots,\ \mathit{not}\ ab_{j_n}(x)$$

belongs to P_Γ.

(b) If there exists a positive path from r to p over the links $j_1, ..., j_n$ then the clause

$$ab_j(x) \leftarrow r(x),\ \mathit{not}\ ab_{j_1}(x), \ldots,\ \mathit{not}\ ab_{j_n}(x)$$

belongs to P_Γ. □

We demonstrate the transformation from Γ into P_Γ in the next two examples.

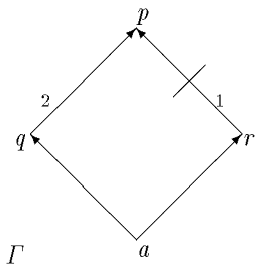

Fig. 4. Nixon-Diamond

Example 3. The corresponding program P_Γ of Γ in the figure 4 is:

$a \rightarrow r$	$r(a) \leftarrow$
$a \rightarrow q$	$q(a) \leftarrow$
$r \not\rightarrow_1 p$	$\neg p(x) \leftarrow r(x),\ \mathit{not}\ ab_1(x)$
	$ab_1(x) \leftarrow p(x)$
$q \rightarrow_2 p$	$p(x) \leftarrow q(x),\ \mathit{not}\ ab_2(x)$
	$ab_2(x) \leftarrow \neg p(x)$

It is easy to see that P_Γ has only two answer sets $\{r(a), q(a), p(a), ab_1(a)\}$ and $\{r(a), q(a), \neg p(a), ab_2(a)\}$. □

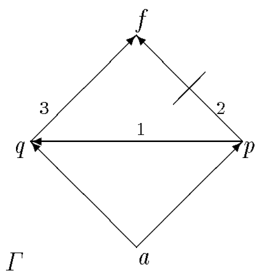

Fig. 5. Penguin-Bird-Fly

Example 4. Let consider the net Γ in figure 5. a denotes Tweety, p, q, and f are penguin, bird, and fly, respectively. The corresponding program P_Γ consists of

$$\begin{array}{ll}
a \rightarrow p & p(a) \leftarrow \\
a \rightarrow q & q(a) \leftarrow \\
p \not\rightarrow_2 f & \neg f(x) \leftarrow p(x),\ not\ ab_2(x) \\
 & ab_2(x) \leftarrow f(x) \\
q \rightarrow_3 f & f(x) \leftarrow q(x),\ not\ ab_3(x) \\
 & ab_3(x) \leftarrow \neg f(x) \\
p \rightarrow_1 q & q(x) \leftarrow p(x),\ not\ ab_1(x) \\
 & ab_1(x) \leftarrow \neg q(x) \\
\text{and} & ab_3(x) \leftarrow p(x),\ not\ ab_1(x)
\end{array}$$

The unique answer set of P_Γ is $\{p(a), q(a), \neg f(a), ab_3(a)\}$. □

The relationship between the answer set semantics of P_Γ and the credulous semantics of Γ can be established in the following way. First, for any set of paths in Γ, E, we define:

$M_E = \{q(a) \mid$ there is a positive path from $a \in I_\Gamma$ to q in E $\} \cup \{\neg q(a) \mid$ there is a negative path from $a \in I_\Gamma$ to q in E $\}$.

The next theorem point out the relationship between the answer set semantics of the program P_Γ and the credulous semantics of the inheritance network Γ.

Theorem 16. *M is an answer set of P_Γ iff there exists a credulous extension E of Γ such that $M_E = M \backslash AB$ where AB denotes the set of all grounded instance of abnormal-predicates in P_Γ.* □

5 Conclusion

We have studied the relationship between semantical concepts of nonmonotonic inheritance and of argumentation framework. In chapter 3 we showed that argumentation framework, a general form of argument-based reasoning can be applied to specify nonmonotonic inheritance reasoning in a simple way. We proved that a consistent and acyclic network can be viewed as an argumentation framework so that the credulous semantics of the former coincides with the stable semantics of the latter.

The capturing of credulous semantics of nonmonotonic inheritance by argumentation framework shows that argumentation can be applied successfully to formulize nonmonotonic inheritance reasoning. It is interesting to notice that many new developed approaches to nonmonotonic reasoning [6, 24, 2] can not applied directly to inheritance reasoning as we did for Dung's argumentation framework. Geffner and Pearl's conditional entailment [6] is too weak as we can not draw the conclusion "Tweety is an animal with feather" if we replace the rule "Bird fly" by two rules "Bird are animals with feather" and "Animal with feather fly" in the Penguin-Bird-Fly example. Simary and Loui's defeasible reasoning [24] gives unintuitive answer even in simple cases as in the Penguin-Bird-Fly example given that Tweety is a penguin and a bird. Delgrande and Schaub's general approach [2] cannot give proper answer in the example with $R = \{a \rightarrow r, a \rightarrow p, a \rightarrow \neg q, p \rightarrow q, q \rightarrow \neg s, q \rightarrow r, r \rightarrow s\}$ (figure 18, page 158 [13]).

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Appendix: Proof of selected theorems

In this section we prove that the credulous semantics of Γ and the stable semantics of the corresponding argumentation framework coincide. At first, we prove some general properties of AFΓ.

Lemma 17. *Let Γ be an acyclic, consistent inheritance network and AF_Γ=<AR_Γ, $attacks_\Gamma$> is the corresponding argumentation framework. Then, if σ attacks σ' then $deg_\Gamma(\sigma) \leq deg_\Gamma(\sigma')$.* □

Proof. Consider three cases:

1. σ attacks σ' by a direct link then it is clear that $deg_\Gamma(\sigma) = deg_\Gamma(\sigma')$ because σ' and σ have the same begin and the same end nodes.
2. σ attacks σ' by conflict. Then, either σ and σ' have the same begin and end and therefore $deg_\Gamma(\sigma) \leq deg_\Gamma(\sigma')$ or there is some prefix of σ' which is conflict with σ, in this case we have $deg_\Gamma(\sigma) < deg_\Gamma(\sigma')$.
3. σ attacks σ' by preemption. Then, there is a prefix $\alpha = \pi(x, \tau, u) \rightarrow y$ (resp. $\pi(x, \tau, u) \not\rightarrow y$) of σ' such that $\sigma = \pi(x, \delta, v, \gamma, u)$ with $v \rightarrow y$ (resp. $v \not\rightarrow y$) in Γ. By definition of deg_Γ we have $deg_\Gamma(\sigma) < deg_\Gamma(\sigma')$.

The lemma is proved from these three cases. □

Lemma 18. *Let S be set of arguments and σ be an argument defendable wrt S. Then, each $\alpha \in pre(\sigma)$ is defendable wrt S.*

Proof. If $\alpha \in pre(\sigma)$ and β is an argument which attacks α then β attacks σ and therefore β is attacked by S. Hence, α is defendable wrt S. □

Lemma 19. *Let S be a set of argument in AF_Γ. If $S \mid\sim \sigma$ then σ is not attacked by S.*

Proof. Assume that there exists an argument τ in S such that $(\tau, \sigma) \in attacks$. By definition of attack we have three cases:

1. τ attacks σ by direct link. Then, σ is preempted in S.
2. τ attacks σ by conflict. Then, σ is conflicted or non constructible in S.
3. τ attacks σ by preemption. Then, σ is preempted or non constructible in S. From the three cases, we learn that if $(\tau, \sigma) \in attacks$ then $S \mid\not\sim \sigma$. Contradictory !!! Thus, S does not attack σ. □

The next lemma follows directly from lemma 19, and the definition of the stable extension.

Lemma 20. *If E is a stable extension of AF_Γ and $\sigma \notin E$. Then, $E \mid\not\sim \sigma$.* □

Further, it is easy to prove the next two lemmae.

Lemma 21. *If E is a stable extension of AF_Γ, then $\Gamma \subset E$, and for every path $\sigma \in E$, σ is not preempted in E.* □

Lemma 22. *Let E be a credulous extension of Γ. Then σ is attacked by E if $\sigma \notin E$.*

Proof. Obviously, σ is a compound path. Without a lost of generality, assume that σ is a positive path $\pi(x, \tau, u) \to y$. Since σ does not belong to E, $E \not\mid\!\!\sim \sigma$. Hence,

1. σ is conflict in E. It means, there is some path α in E that is conflict with σ. Therefore, σ is attacked by E.
2. σ is not constructible in E. There is a $\alpha \in pre(\sigma)$ such that all proper prefix of α is contained in E but not α. Therefore $E \not\mid\!\!\sim \alpha$. Further, since α is constructible in E. For we have two sub-cases:
   α is conflicted in E. Similarly to the first case, α is attacked by E. Thus, σ is attacked by E.
   α is preempted in E. See next case.
3. σ is preempted in E. That is, there is some node v with $v \not\to y \in \Gamma$ and there is some $\delta = \pi(x, \alpha, v, \beta, u)$ in E. Thus, δ attacks σ.

From the three cases, we can conclude that σ is attacked by E. □

From the lemmae 17–22 we can prove the theorem 13:

Proof of Theorem 13

Proof. 1. '$\longleftarrow$' Suppose that E is a stable extension of AF_Γ. Let $\sigma \notin E$. From lemma 19 we have $E \not\mid\!\!\sim \sigma$ (i). Now, let $\sigma \in E$. Any prefix of σ is defendable wrt E (Lemma 18). Thus, all prefixes of σ are containing in E. So, σ is constructible and non-conflicted in E. Further, σ is not preempted in E (Lemma 21). Hence, σ is defeasibly inheritable in E (ii). From (i) and (ii) we can conclude that E is a credulous extensions of Γ.

2. '$\longrightarrow$' Now, if E is a credulous extension of Γ then E is conflict-free (Lemma 18). It is easy to see that E attacks every argument which does not belong to E (Lemma 22). Hence, E is a stable extension of AF_Γ.
   The theorem is proved. □