

# Viewing in 3D

# Steps

- Clip against view volume
- Projection on projection plane
- Transform to viewport in 2D device coordinates for display (**previous chapter**)

# Projection

- This chapter: 3D objects onto 2D plane
- **Definition:** The projection of a 3D object is defined by straight projection rays (*projectors*) emanating from the *center of projection*, passing through each point of the object, intersecting a *projection plane* to form the projection.

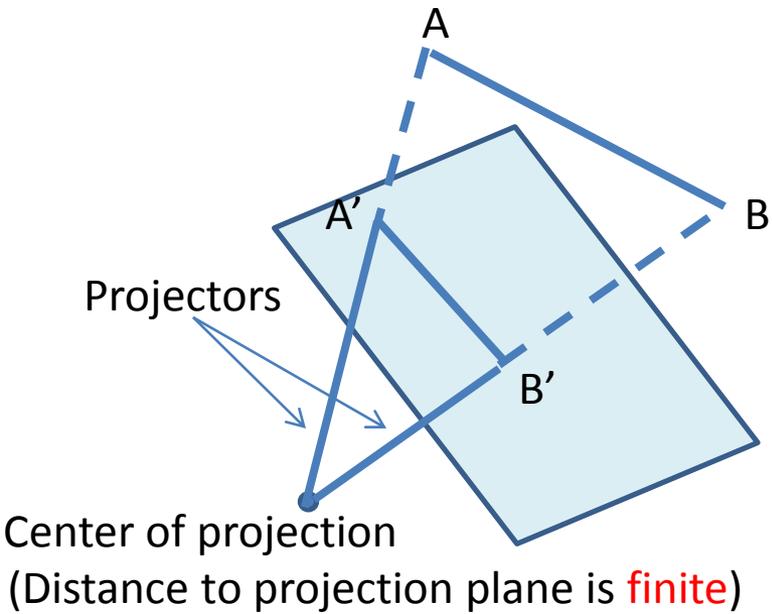
# What should be known before a projection?

- Object
  - Projection plane (could also be surface)
  - Projectors (could be curve)
  - Center of projection
- 
- We study: Planar geometric projections (projection plane and lines as projectors)

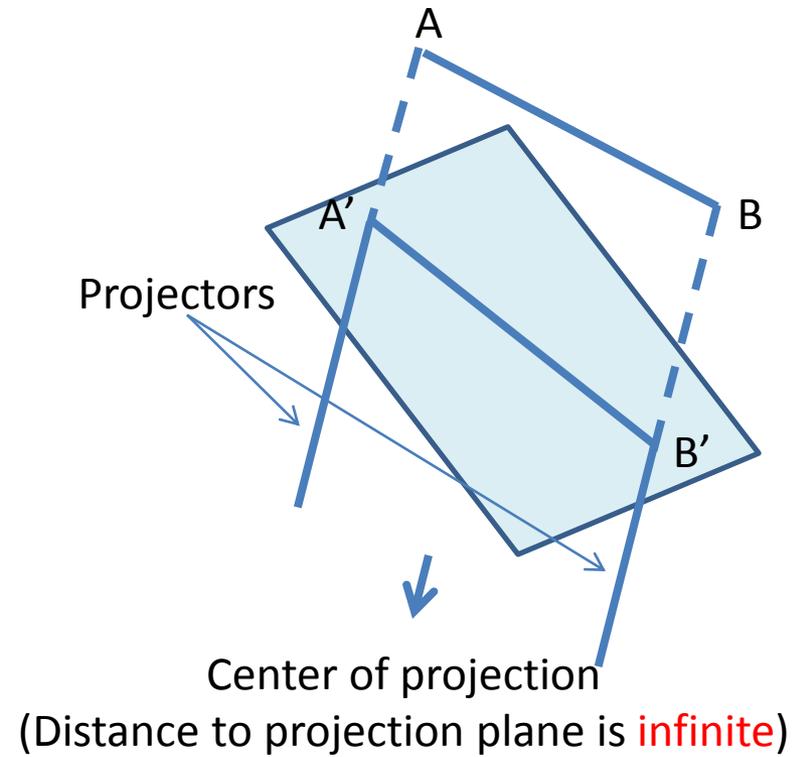
# What should be known before a planar geometric projection?

- Object
- Projection plane
- Projectors
- Center of projection
  - At finite: a given point with finite coordinates (**perspective projection**)
  - At infinite: the point is at infinite (**parallel projection**)

# Example



Perspective Projection



Parallel Projection

# Specification of a Projection

- Object
- Plane
- Center of projection
  - Perspective: The point (center)  $[x \ y \ z \ 1]^T$
  - Parallel: The direction  $(a, b, c, 0)$

# Properties of Projection

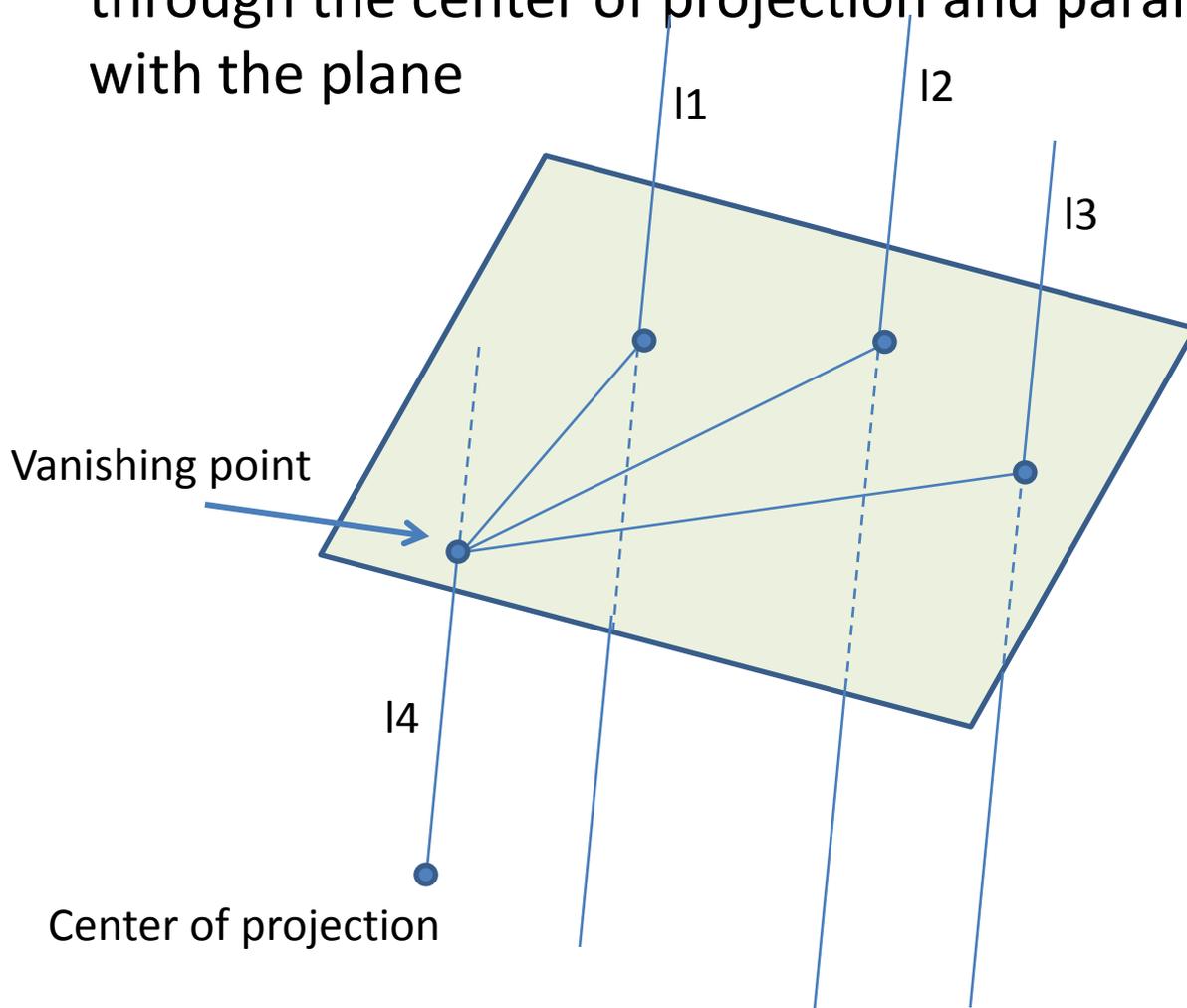
- Perspective projection: realistic view of objects (*perspective foreshortening*) but
  - Size: changes with the distance from object to center of projection
  - Shape, measurements are lost
  - Angles are maintained only if it is parallel with the projection plane
  - Parallel lines are also not maintained
- Parallel projection: less realistic
  - Measurements and parallelism are maintained
  - Angles are maintained only if it is parallel with the projection plane

# Perspective Projection

- The perspective projections of any set of parallel lines that are not parallel to the projection plane converge to a *vanishing point*
- There is no vanishing point if the lines are parallel to the projection plane
- How do we determine the vanishing point?

# Perspective Projection

- Vanishing point is the intersection point of the line that goes through the center of projection and parallel to the given lines with the plane

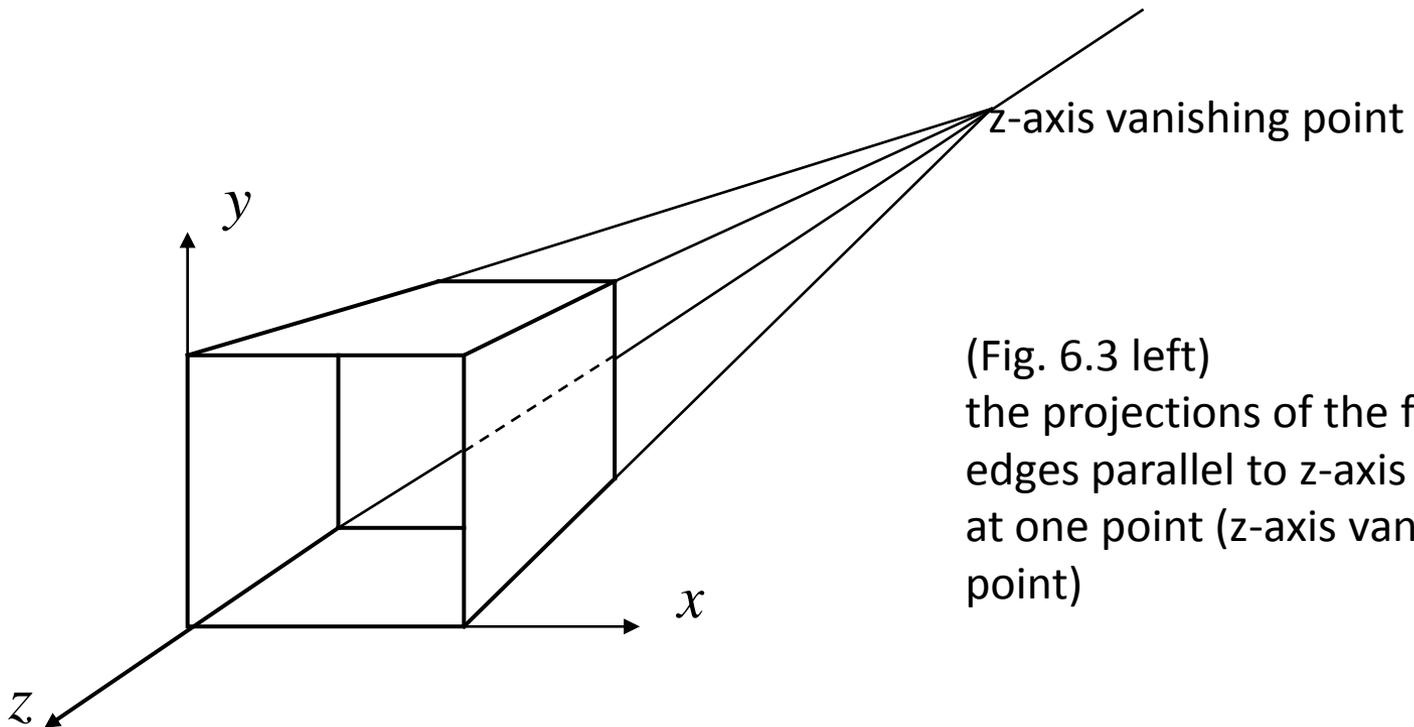


# Principal vanishing point

- If the set of lines parallel to one of the axis => the vanishing point is called an *axis vanishing point* (or *principal vanishing point*)
- There are at most three axis vanishing points
- Perspective projection is classified by the number of principal vanishing points (one, two, and three)

# Principal vanishing points

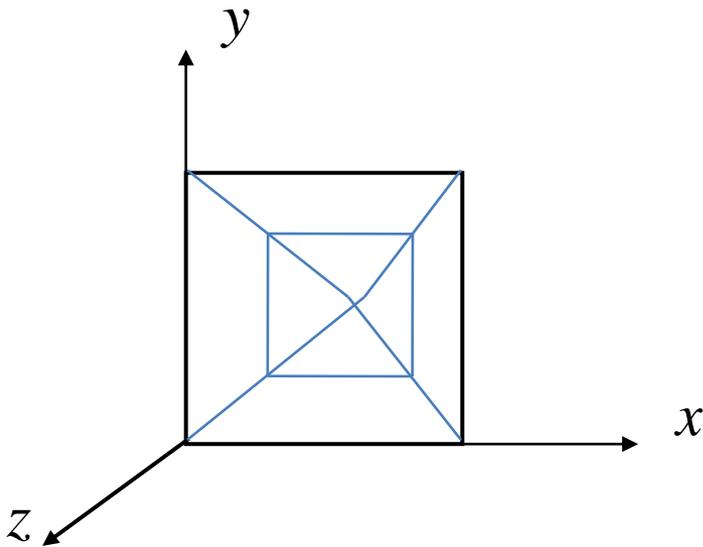
- Perspective projection of a unit cube onto a plane perpendicular to the z-axis → projections of edges parallel to the y-axis will meet at vanishing point (on z-axis)
- Projections of edges parallel to the x-axis and y-axis do not meet



(Fig. 6.3 left)  
the projections of the four  
edges parallel to z-axis meet  
at one point (z-axis vanishing  
point)

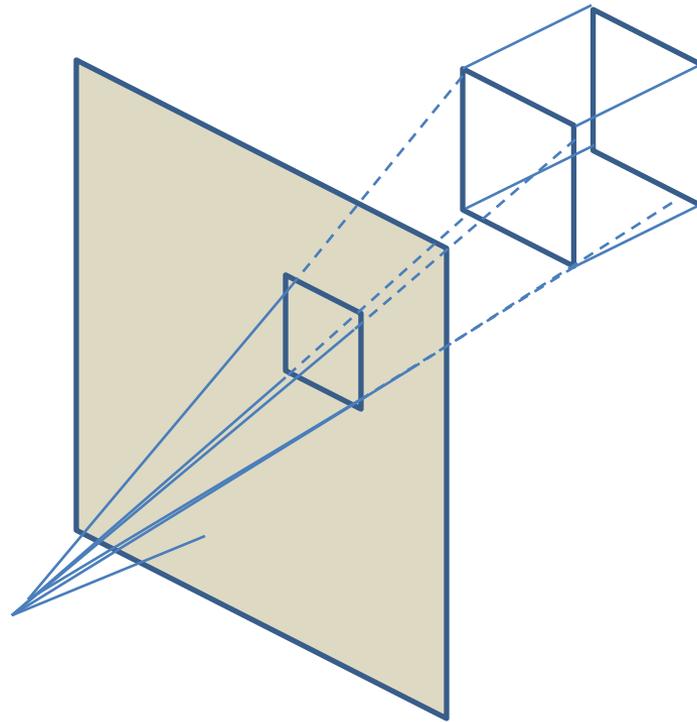
# Vanishing Point

- Perspective projection of a unit cube onto a plane perpendicular to the z-axis → projections of edges parallel to the y-axis will meet at vanishing point (on z-axis)
- Projections of edges parallel to the x-axis and y-axis do not meet



(Fig. 6.3 right)  
the projections of the four  
edges parallel to z-axis  
meet at one point

# Example of one-point perspective projection

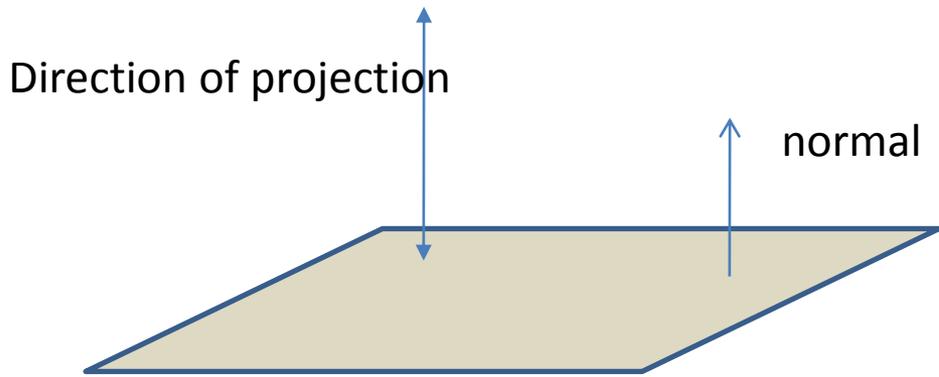


Board

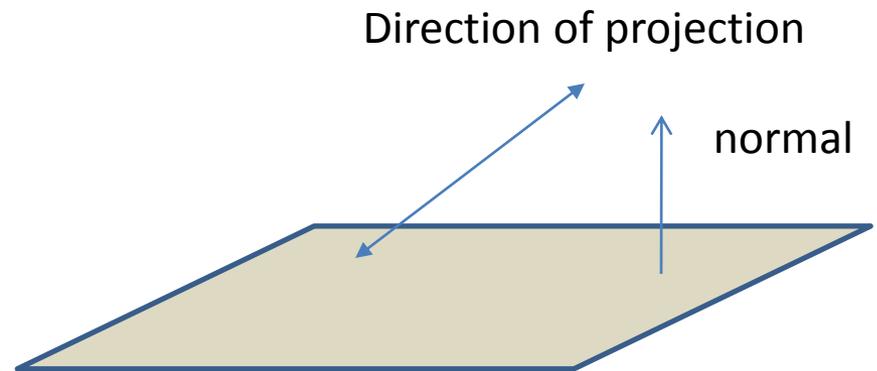
# Example of two-point perspective projection

# Parallel projection

- Classified by the relation of the direction of projection and the normal of the projection plane

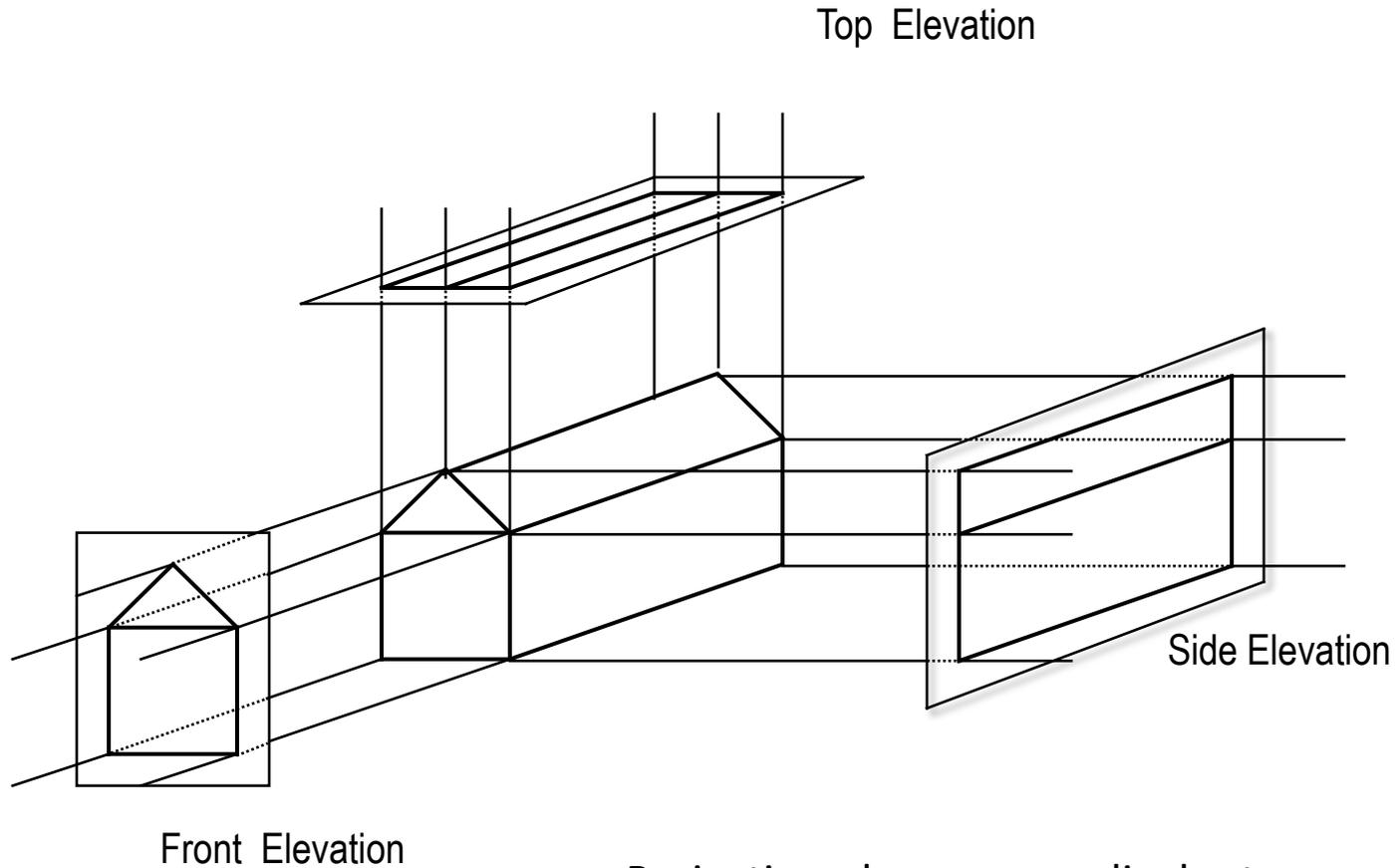


*Orthographic projection*



*Oblique projection*

# 3 Most common types of orthographic Projections

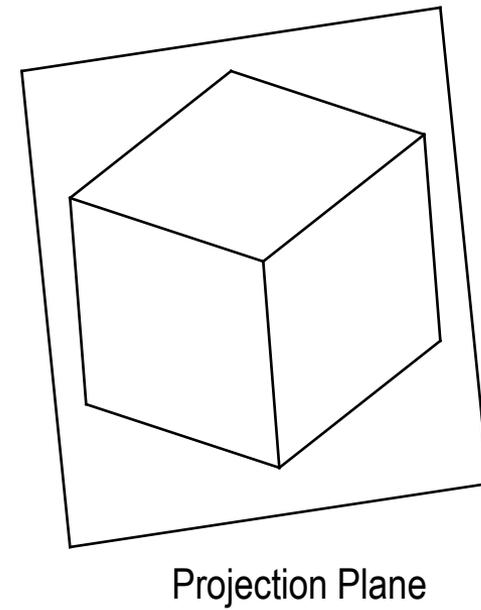
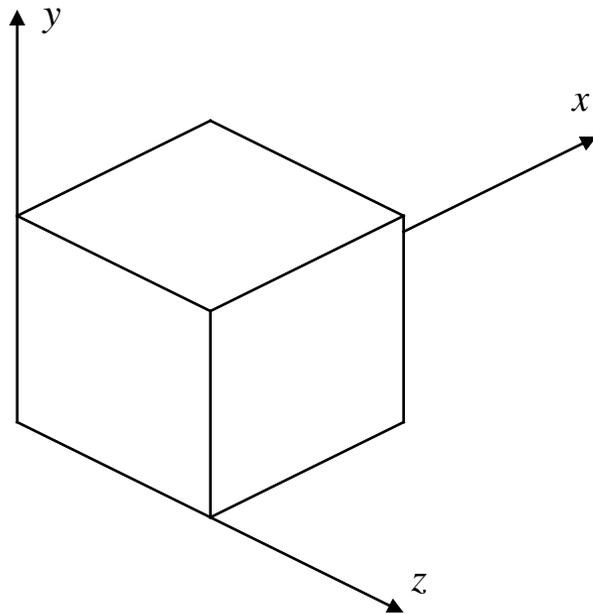
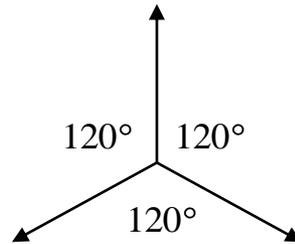


Projection plane perpendicular to one axis

# Axonometric orthographic projections

- Projection plane not normal to any principal axis
  - Several faces of object are shown at once
  - Foreshortening is uniform
  - Parallelism is preserved but not angles
- *Isometric projection* is most common: projection plane normal makes equal angle with each principal axis ( $|dx|=|dy|=|dz|$ )
  - All principal axis are equally foreshortened
  - All principal axis make equal angles one with other

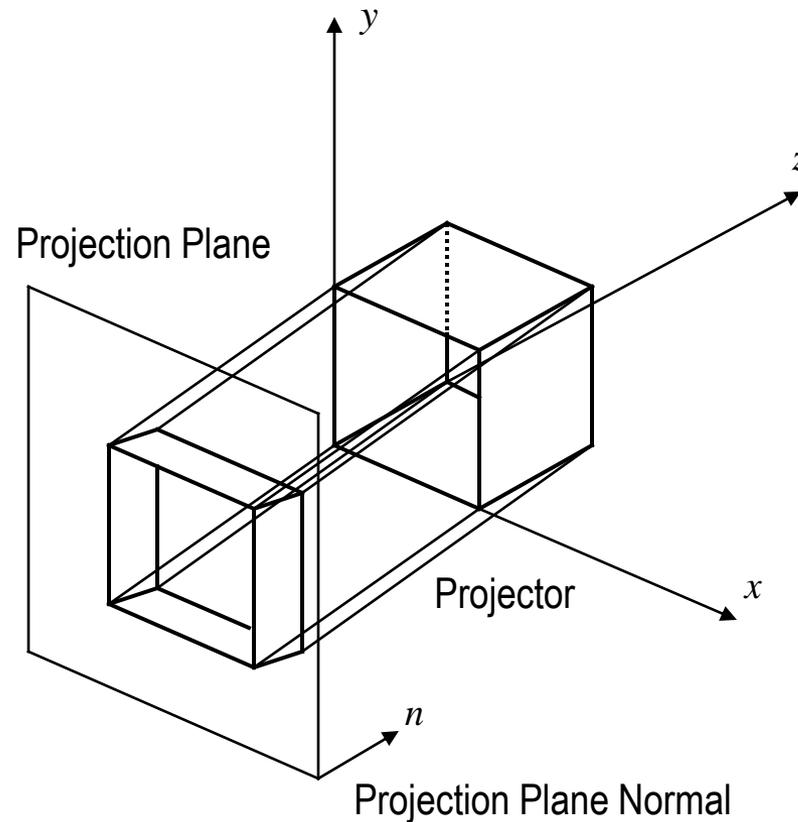
# Isometric Projection

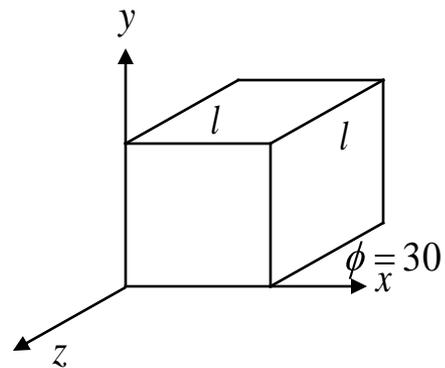
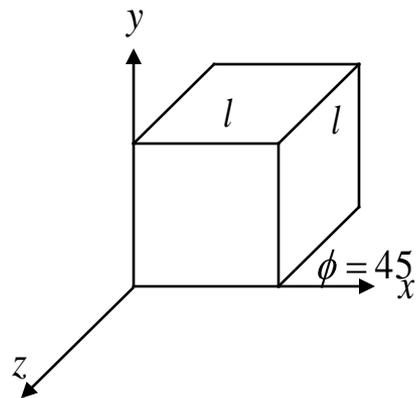


# Oblique projection

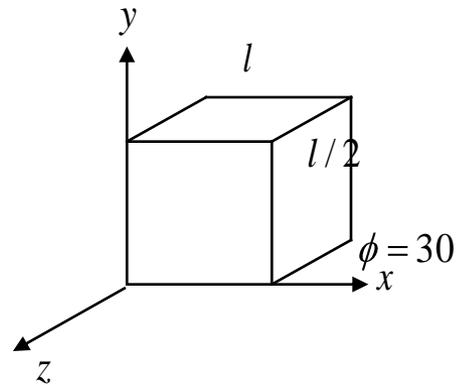
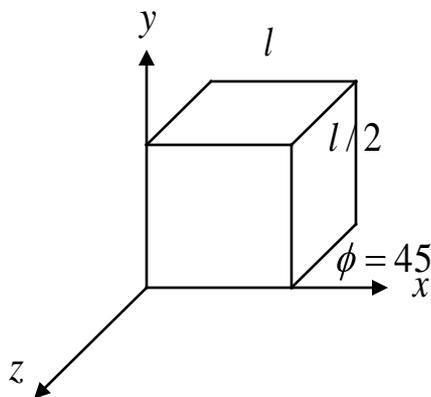
- Projection plane is normal to a principal axis
  - Faces parallel to this plane maintain measurements
- Frequently used
  - Cavalier: direction of projection makes 45 degree angle with the projection plane
  - Cabinet: direction of projection makes an angle  $X$  such that  $\arctan(2)=63.4$  degree

# Oblique Projection





Cavalier projection of unit cube onto plane  $z = 0$ .



Cabinet projection of unit cube onto plane  $z = 0$ .

Why is  $\phi = 45$

# Taxonomy of Geometric Projection

