Viewing in 3D

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1 Projections

Two different types of projections: *parallel* and *perspective*. Specified by COP *center of projection* and *projection plane* (PP). The distance between COP and PP decides the type of projection: finite is perspective, infinite is parallel. A hierarchy of commonly used projection is given.

2 Specifying a 3D View

- **VRP**: view reference point

- **VPN**: view plane normal (together with VRP specified the view plane)

- **VRC**: view reference coordinate system (specified by the origin (at VRP), the VPN (n-axis), u-axis (VUP), v-axis where u,v-axes on the view plane and u,v,n-axes creates a right-handed coordinate system)

- **Window**: specified by two points (u_{min}, v_{min}) and (u_{max}, v_{max}), **CW** is the center of the window.

- **PRP**: projection reference point — given in the VRC coordination system (not the world-coordinate system). If the projection type is perspective then the PRP is the COP. If the projection type is parallel then DOP is specified by the PRP and the CW.

- **View Volume**: specified by the PRP and the Window (perspective: pyramid with apex at PRP and edges from PRP to corners of Window; parallel: paprallelepiped with sides parallel to the DOP and passing through corners of Window).

- **Back/Front Clipping plane**: used to limit the view volume, normal is the same as VPN, specified by the front and back distance respectively.

3 Computing Projection

Given a 3D view speficied by the parameters:

- the view plane is given by $z = z_p$

- the COP is given by the unit vector (d_x, d_y, d_z) from the point $(0, 0, z_p)$ and the distance Q from $(0, 0, z_p)$

We want to determine the projection of a point $P(x, y, z)$ onto the projection plane. This can be obtained through the matrix:

$$M_{general} = \begin{bmatrix} 1 & 0 & -\frac{d_x}{d_z} & z_p\frac{d_x}{d_z} \\ 0 & 1 & -\frac{d_y}{d_z} & z_p\frac{d_y}{d_z} \\ 0 & 0 & -\frac{z_p}{Qd_z} & \frac{z_p^2}{Qd_z} + z_p \\ 0 & 0 & \frac{1}{Qd_z} & \frac{z_p}{Qd_z} + 1 \end{bmatrix}$$

This matrix can be specialized in different cases:

- $Q = \infty$, $z_p = 0$, $(d_x, d_y, d_z) = (0, 0, -1)$: this is orthographic projection onto the xy-plane

- $Q = d$, $z_p = d$, $(d_x, d_y, d_z) = (0, 0, -1)$: this is perspective projection with COP at the origin, viewing opposite the positive z-direction, and the view plane is parallel to the xy-plane and is of the distance d to the xy-plane.

- $Q = d$, $z_p = 0$, $(d_x, d_y, d_z) = (0, 0, -1)$: this is perspective projection with COP at $(0, 0, -d)$, viewing opposite the positive z-direction, and the view plane is parallel to the xy-plane and is of the distance d to the xy-plane.

- $Q = \infty$, $z_p = 0$, $(d_x, d_y, d_z) = (cos\alpha, sin\alpha, -1)$, this is a cavalier projection (as given by the Figure 6.10)

- $Q = \infty$, $z_p = 0$, $(d_x, d_y, d_z) = (\frac{cos\alpha}{2}, \frac{sin\alpha}{2}, -1)$, this is a cabinet projection (as given by the Figure 6.11)

4 Implementing Planar Geometric Projections

Before we can implement the projections, we need to clip against the view volume. The process starts with transforming an arbitrary view volume to *canonical view volume*, specified by

- for parallel projection: the canonical view volume is specifed by the bounding planes: $x = -1$, $x = 1$, $y = -1$, $y = 1$, $z = 0$, $z = -1$ (in this case: a box containing points $P(x, y, z)$ such that $-1 \leq x \leq 1, -1 \leq y \leq 1, -1 \leq z \leq 0$).

- for perspective projection: the canonical view volume is specifed by the planes: $x = -z$, $x = z$, $y = -z$, $y = z$, $z = -z_{min}$, $z = -1$

Transformation of arbitrary view volume to canonical view volume:

- Parallel projection: N_{par} is the result of the following sequence of transformations:

    1. Translate VRP to the origin
    2. Rotate VRC such that the VPN becomes the z-axis, the u-axis becomes the x-axis, and the v-axis becomes the y-axis
    3. Shear such that the direction of the projection becomes parallel to the z-axis
    4. Translate and scale into the parallel-projection canonical view volume

- Perspective projection: N_{per} is the result of the following sequence of transformations:

    1. Translate VRP to the origin
    2. Rotate VRC such that the VPN becomes the z-axis, the u-axis becomes the x-axis, and the v-axis becomes the y-axis
    3. Translate such that COP (PRP) is at the origin
    4. Shear such that the center line of view volume becomes the z-axis
    5. Scale so that the view volume becomes canonical view volume