

Homework 3

Artificial Intelligence II — CS575

October 5, 2007

This is a reading homework. Please download the paper:

C. Baral, M. Gelfond, “Logic Programming and Knowledge Representation”, Journal of Logic Programming, 19,20:73-148, 1994. (Survey paper) from the Web site: <http://www.cs.ttu.edu/~mgelfond/>

Please do the following:

1. Read **at least** the first 12 pages of the paper.
2. Write a two pages report (12-10 point font only!) in which you

- explain the following notion:

(a) Stable model

The notion of stable model is defined first for programs without negation as failure literals (positive programs) and then for programs with negation as failure literals (general programs).

– A simple way:

Let P be a positive program and P does not contain any variable. A Herbrand *model* of P is a set of atoms X from the Herbrand base of P such that for every rule $r \in P$, if $pos(r) \subseteq X$ then $head(r) \in X$. M_P , the minimal Herbrand model of P , is defined as the stable model of P .

Let P be a positive program with variables. Let $ground(P)$ be the set of all ground instantiations of rules in P . The stable model of $ground(P)$ is defined as the stable model of P . That is, $M_{ground(P)}$ is the stable model of P .

– A compact way:

If P is a positive program. A Herbrand *model* of P is a set of atoms X from the Herbrand base of P such that for every ground rule $r \in ground(P)$, if $pos(r) \subseteq X$ then $head(r) \in X$. The minimal Herbrand model of P is defined as the stable model of P .

For a general program P let $ground(P)$ be the set of all ground instantiations of rules in P . Let X be a set of atoms from the Herbrand base of P . We define P^X to be the program obtained from $ground(P)$ by

- i. removing all rule $r \in ground(P)$ such that the body of r contains a literal *not* a and $a \in X$; and
- ii. removing all atoms of the form *not* a from the remaining rules.

A set of atoms S from the Herbrand base of P is a stable model of P if S is the stable model of P^S .

Example 0.1 For the program P

$$\begin{aligned} p &\leftarrow \text{not } q \\ q &\leftarrow \text{not } p \end{aligned}$$

we have $P^{\{p\}}$ consists of only the fact p . Thus, $\{p\}$ is a stable model of P . We can also check that $\{q\}$ is another stable model of P .

(b) Categorical program

A program P is a categorical program if it has a unique stable model. The program in Example 0.1 is not a categorical program but the program consisting of a single fact

$$p \leftarrow$$

is a categorical program.

It is easy to see that every positive program is a categorical program.

(c) Stratified program

To define stratified programs, we need the notion of $atoms(p)$ which denotes the set of all ground atoms whose predicate is p .

A program P is stratified if it has a stratification.

A partition π_0, \dots, π_n of the set of predicates of P is called a stratification if for every predicate $p \in \pi_s$ and every ground rule

$$a_0 \leftarrow a_1, \dots, a_n, not\ b_1, \dots, not\ b_m$$

of P such that $a_0 \in atoms(p)$ the following conditions are satisfied:

- i. for every $1 \leq i \leq n$, the predicate of atoms a_i belongs to p_l for some $l \leq s$; and
- ii. for every $1 \leq i \leq m$, the predicate of atoms b_i belongs to p_l for some $l < s$.

Example 0.2 The program P

$$\begin{aligned} p(X) &\leftarrow q(X), not\ r(X) \\ r(Y) &\leftarrow q(X) \\ r(a) &\leftarrow \\ q(b) &\leftarrow \end{aligned}$$

is a stratified program since we can have the stratification $\{q, r\}, \{p\}$.

On the other hand, the program in Example 0.1 is not a stratified program since none of the possible partitions of $\{p, q\}$ satisfying the above two conditions.

For each notion, please give an example to illustrate it.

- formulate the following information

Love is a relationship between two persons. Normally, it is symmetric. Normally, parents love their children. Math is a parent of Physics. Physics loves Biology.

as a logic program. Makes sure that your program can be used to confirm that Physics loves Math and Biology. Compute the stable models of your program. Is your program categorical? Is it stratified?

We use $love(X, Y)$ to denote X loves Y ; $parent(X, Y)$ to denote X is a parent of Y . We can come up with the following program P

$$\begin{aligned} love(Y, X) &\leftarrow love(X, Y), not\ ab(love, X, Y) \\ love(X, Y) &\leftarrow parent(X, Y), not\ ab(parent, X, Y) \\ parent(math, physics) &\leftarrow \\ love(physics, biology) &\leftarrow \end{aligned}$$

where $ab(love, X, Y)$ denotes that the “love relationship” between X and Y is “abnormal” and hence is not symmetric; similarly, $ab(parent, X, Y)$ denotes that the parental relationship between X and Y is abnormal.

To compute the stable model of this program, first one needs to determine $ground(P)$. The ground program consists of the facts and all possible combinations of X and Y for the first two rules (which means

that we have 9 ground rules for each rule). For example, for $X = \textit{math}$ and $Y \in \{\textit{math}, \textit{physics}, \textit{biology}\}$, we get the following ground rules from the first rule:

$$\begin{aligned} \textit{love}(\textit{math}, \textit{physics}) &\leftarrow \textit{love}(\textit{physics}, \textit{math}), \textit{not } \textit{ab}(\textit{love}, \textit{math}, \textit{physics}) \\ \textit{love}(\textit{math}, \textit{math}) &\leftarrow \textit{love}(\textit{math}, \textit{math}), \textit{not } \textit{ab}(\textit{love}, \textit{math}, \textit{math}) \\ \textit{love}(\textit{math}, \textit{biology}) &\leftarrow \textit{love}(\textit{biology}, \textit{math}), \textit{not } \textit{ab}(\textit{love}, \textit{math}, \textit{biology}) \end{aligned}$$

Now, notice that there is no rule in the program that has $\textit{ab}(\dots)$ as the head. This implies that no stable model of the program would contain any atom of the form $\textit{ab}(\dots)$. Hence, for every stable model S of the program, the program P^S will contain the following rules

$$\begin{aligned} \textit{love}(Y, X) &\leftarrow \textit{love}(X, Y) && \text{for every combination of } X \text{ and } Y \\ \textit{love}(X, Y) &\leftarrow \textit{parent}(X, Y) && \text{for every combination of } X \text{ and } Y \\ \textit{parent}(\textit{math}, \textit{physics}) &\leftarrow \\ \textit{love}(\textit{physics}, \textit{biology}) &\leftarrow \end{aligned}$$

The stable model of this program is

$$\left\{ \begin{array}{l} \textit{parent}(\textit{math}, \textit{physics}), \textit{love}(\textit{math}, \textit{physics}), \\ \textit{love}(\textit{physics}, \textit{math}), \textit{love}(\textit{physics}, \textit{biology}), \textit{love}(\textit{biology}, \textit{physics}) \end{array} \right\}$$

Together with the above argument, we conclude that the above is the unique stable model of P .

As P has a unique stable model it is categorical.

It is easy to see that $\{\textit{parent}, \textit{ab}\}, \{\textit{love}\}$ is a partition of the set of predicates in P . Thus, the program is stratified.