Solution to Homework 7

November 7, 2006

Problem 1. We have

$$AB^+_{\mathcal{F}} = ABCD$$
$$BC^+_{\mathcal{F}} = ABCD$$
$$CD^+_{\mathcal{F}} = ABCD$$
$$AD^+_{\mathcal{F}} = ABCD$$

Thus

a) non-trivial functional dependencies are $AB \to C$, $AB \to D$, $BC \to A$, $BC \to D$, $AD \to B$, $AD \to C$, $CD \to A$, $CD \to B$, and some more e.g., $ABD \to C$, $ABC \to D$, $BCD \to A$, etc.

b) AB, BC, CD, and AD are the keys of $\mathbf{R}$.

c) superkeys of $\mathbf{R}$ that are not keys are ABC, ABD, BCD, ACD, and $ABCD$.

Problem 2.

a) $\mathbf{R}$ is in BCNF because all non-trivial dependencies are of the form $\bar{X} \to \bar{Y}$ where $\bar{X}$ is a superkey.

b) No decomposition is needed as $\mathbf{R}$ is already in BCNF.

Problem 3. The projection of the set of functional dependencies $\mathcal{F}$ on the set of attributes $\{A, B, C\}$ is

$$\pi_{ABC}(\mathcal{F}) = \{\bar{X} \to \bar{Y} \mid \bar{X} \to \bar{Y} \in \mathcal{F}^+ \text{ and } \bar{X} \cup \bar{Y} \subseteq ABC\} =$$

$$\{AB \to C, BC \to A\}$$