Warmup — Homework 1

August 31, 2005

Due: 11:59 pm — August 30, 2005

Submit to "Assignment 1" in the homework submission website.

1. Assume that we have four propositions A, B, C, and D. How many models are there for the following sentences?

(a) $(A \vee B) \wedge (B \wedge (D \vee C))$

Any model of this formula has to have B true. The truth assignment for A can be done arbitrarily. There are three possibilities for D and C so that $D \vee C$ is true. So, there is a total of 6 models for this formula.

(b) $A \vee C$

Here, there are three possibilities for A and C so that $A \vee C$ is true. The truth assignment for B and D does not matter. So, there is 12 models for this formula.

(c) $A \Leftrightarrow C \Leftrightarrow D$

This requires that A, C, and D have the same truth value (2 possibilities). For B, there is two. So, this formula has four models.

2. Assume that we have the theory $T = \{A \vee B,\ C \wedge D, (A \wedge C) \Rightarrow (B \wedge D)\}$.

(a) Is there any model for T in which B is false?

No. B is false so A must be true. This will imply that $(A \wedge C) \Rightarrow (B \wedge D)$ is false, i.e., no model of T can have B false.

(b) Use the model theoretic approach to show that the sentence $D \Rightarrow (A \vee B)$ is true.

From (a), we know that B must be true. By definition of T, C and D must be true. This results in that T has the following models:

A	B	C	D	$A \vee B$	$C \wedge D$	$(A \wedge C) \Rightarrow (B \wedge D)$	$D \Rightarrow (A \vee B)$
t	t	t	t	t	t	t	t
f	t	t	t	t	t	t	t

This shows that the therory entails $D \Rightarrow (B \wedge D)$.

(c) Verify your proof in (b) by using the inference rules to show that the sentence $D \Rightarrow (A \vee B)$ is true.

Using the rule $\frac{(\alpha \wedge \beta)}{\alpha}$ and the fact that $C \wedge D$ is a part of T, we conclude D.

Using the rule $\frac{(\alpha, \beta)}{\alpha \Rightarrow \beta}$ and the fact that $A \vee B$ is a part of T, we conclude $D \Rightarrow (A \vee B)$.