Given a relation schema BCDHF with the set of FDs $\{BG \rightarrow CD, G \rightarrow F, CD \rightarrow GH, C \rightarrow FG, F \rightarrow D\}$. Find a BCNF decomposition of the schema.

First, we compute some closures:

- $BG^+ = BGCDFH$ (remember that $BGCDFH$ is a short hand for the set $\{B, G, C, D, F, H\}$);
- $G^+ = GFD$;
- $CD^+ = CDGHF$;
- $C^+ = CFGDH$;
- $F^+ = FD$

To identify BCNF violations, we need to know the keys of the schema. Since B does not occur in the right hand side (RHS) of any FD, we conclude that B must be part of any key of the schema. Furthermore, since $B^+ = B$, we conclude that any key of the schema must have at least two attributes.

From the above computation, we can conclude that BG and BC are keys of the schema. Computing the closures of BD, BH, and BF we can determine that they are not keys of the schema. Notice also that in order to have G in the closure we need either G or C. Therefore, BG and BC are the only two keys of the schema.

Now that we have the keys, we would like to check the given FDs for BCNF violations. Clearly, all but the first FD violate the BCNF condition since of them is a non-trivial FD whose left hand side is not a superkey.

We select $G \rightarrow F$ and decompose the given schema into two shema $R_1 = (GF, \{G \rightarrow F\})$ and $R_2 = (BCDGH, \{BG \rightarrow CDH, G \rightarrow D, CD \rightarrow GH, C \rightarrow GDH\})$.

NOTE: The FDs include in the set of FDs of R_1 and R_2 are derived from the closures that we have computed above.

R_1 is already in BCNF because G is now the key of R_1. R_2 is still not in BCNF because its keys are BC and BG (Why?) and the three FDs $G \rightarrow D, CD \rightarrow GH, C \rightarrow GDH$ violate the BCNF condition. Selecting $G \rightarrow D$, we decompose R_2 to $R_3 = (GD, \{G \rightarrow D\})$ and $R_4 = (BDCH, \{BG \rightarrow CH, C \rightarrow GH\})$. The latest schema is still not in BCNF, because $C \rightarrow GH$ violates the BCNF condition. Decompose R_4 using this FD, we have $R_5 = (CGH, \{C \rightarrow GH\})$ and $R_6 = (BC, \{\})$, both are in BCNF.

We get the following BCNF decomposition: R_1, R_3, R_5, R_6.

NOTE: I think I used a different list of FDs in my decomposition in this note.