Relational Normalization Theory

Chapter 8

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Limitations of E-R Designs

- Provides a set of guidelines, does not result in a unique database schema
- Does not provide a way of evaluating alternative schemas
- Normalization theory provides a mechanism for analyzing and refining the schema produced by an E-R design

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Redundancy

- Dependencies between attributes cause redundancy
  - Ex. All addresses in the same town have the same zip code

SSN	*Name*	*Town*	*Zip*
1234	Joe	Stony Brook	11790
4321	Mary	Stony Brook	11790
5454	Tom	Stony Brook	11790
	…………………		

Redundancy

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Redundancy and Other Problems

- Set valued attributes in the E-R diagram result in multiple rows in corresponding table
- Example: Person (*SSN, Name, Address, Hobbies*)
  - A person entity with multiple hobbies yields multiple rows in table Person
    - Hence, the association between *Name* and *Address* for the same person is stored redundantly
  - *SSN* is key of entity set, but (*SSN, Hobby*) is key of corresponding relation
    - The relation Person can't describe people without hobbies

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Example

ER Model

SSN	*Name*	*Address*	*Hobby*
1111	Joe	123 Main	{biking, hiking}

Relational Model

SSN	*Name*	*Address*	*Hobby*
1111	Joe	123 Main	biking
1111	Joe	123 Main	hiking
	…………….		

Redundancy

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Anomalies

- Redundancy leads to anomalies:
  - **Update anomaly**: A change in *Address* must be made in several places
  - **Deletion anomaly**: Suppose a person gives up all hobbies. Do we:
    - Set Hobby attribute to null? No, since *Hobby* is part of key
    - Delete the entire row? No, since we lose other information in the row
  - **Insertion anomaly**: *Hobby* value must be supplied for any inserted row since *Hobby* is part of key

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Decomposition

- **Solution**: use two relations to store Person information
  - Person1 (*SSN, Name, Address*)
  - Hobbies (*SSN, Hobby*)
- The decomposition is more general: people with hobbies can now be described
- No update anomalies:
  - Name and address stored once
  - A hobby can be separately supplied or deleted

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Normalization Theory

- Result of E-R analysis need further refinement
- Appropriate decomposition can solve problems
- The underlying theory is referred to as *normalization theory* and is based on *functional dependencies* (and other kinds, like *multivalued dependencies*)

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Functional Dependencies

- **Definition:** A *functional dependency* (FD) on a relation schema **R** is a constraint $\boldsymbol{X} \rightarrow \boldsymbol{Y}$, where X and Y are subsets of attributes of **R.**
- **Definition**: An FD $\boldsymbol{X} \rightarrow \boldsymbol{Y}$ is *satisfied* in an instance **r** of **R** if for every pair of tuples, t and s: if t and s agree on all attributes in X then they must agree on all attributes in Y
  - Key constraint is a special kind of functional dependency: all attributes of relation occur on the right-hand side of the FD:
    - *SSN* → *SSN, Name, Address*

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Functional Dependencies

- *Address* → *ZipCode*
  - Stony Brook's ZIP is 11733
- *ArtistName* → *BirthYear*
  - Picasso was born in 1881
- *Autobrand* → *Manufacturer, Engine type*
  - Pontiac is built by General Motors with gasoline engine
- *Author, Title* → *PublDate*
  - Shakespeare's Hamlet published in 1600

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Functional Dependency - Example

- Brokerage firm allows multiple clients to share an account, but each account is managed from a single office and a client can have no more than one account in an office
  - HasAccount (*AcctNum, ClientId, OfficeId*)
    - keys are (*ClientId, OfficeId*), (*AcctNum, ClientId*)
  - *Client, OfficeId* → *AcctNum*
  - *AcctNum* → *OfficeId*
    - Thus, attribute values need not depend only on key values

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Entailment, Closure, Equivalence

- **Definition**: If F is a set of FDs on schema **R** and f is another FD on **R**, then F *entails* f if every instance **r** of **R** that satisfies every FD in F also satisfies f
  - Ex: $F = \{A \rightarrow B, B \rightarrow C\}$ and f is $A \rightarrow C$
    - If *Streetaddr* → *Town* and *Town* → *Zip* then *Streetaddr* → *Zip*
- **Definition**: The *closure* of F, denoted F^+, is the set of all FDs entailed by F
- **Definition**: F and G are *equivalent* if F entails G and G entails F

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Entailment (cont'd)

- Satisfaction, entailment, and equivalence are ***semantic*** concepts – defined in terms of the actual relations in the "real world."
  - They define *what these notions are*, **not** how to compute them
- How to check if F entails f or if F and G are equivalent?
  - Apply the respective definitions for all possible relations?
    - *Bad idea*: might be infinite number for infinite domains
    - Even for finite domains, we have to look at relations of *all* arities
  - **Solution**: find algorithmic, *syntactic* ways to compute these notions
    - *Important*: The syntactic solution must be "correct" with respect to the semantic definitions
    - Correctness has two aspects: *soundness* and *completeness* – see later

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Armstrong's Axioms for FDs

- This is the *syntactic* way of computing/testing the various properties of FDs

- **Reflexivity**: If $Y \subseteq X$ then $X \rightarrow Y$ (trivial FD)
  – *Name, Address* $\rightarrow$ *Name*
- **Augmentation**: If $X \rightarrow Y$ then $XZ \rightarrow YZ$
  – If *Town* $\rightarrow$ *Zip* then *Town, Name* $\rightarrow$ *Zip, Name*
- **Transitivity**: If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

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Soundness

- Axioms are *sound*: If an FD f: $X \rightarrow Y$ can be derived from a set of FDs F using the axioms, then f holds in every relation that satisfies every FD in F.
- Example: Given $X \rightarrow Y$ and $X \rightarrow Z$ then

$X \rightarrow XY$	*Augmentation by X*
$YX \rightarrow YZ$	*Augmentation by Y*
$X \rightarrow YZ$	*Transitivity*

  – Thus, $X \rightarrow YZ$ is satisfied in every relation where both $X \rightarrow Y$ and $X \rightarrow Y$ are satisfied
    - Therefore, we have derived the ***union rule*** for FDs: we can take the union of the RHSs of FDs that have the same LHS

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Completeness

- Axioms are *complete:* If F entails f, then f can be derived from F using the axioms
- A consequence of completeness is the following (naïve) algorithm to determining if F entails f:
  – Algorithm: Use the axioms in all possible ways to generate F^+ (the set of possible FD's is finite so this can be done) and see if f is in F^+

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Correctness

- The notions of *soundness* and *completeness* link the syntax (Armstrong's axioms) with semantics (the definitions in terms of relational instances)
- This is a precise way of saying that the algorithm for entailment based on the axioms is "correct" with respect to the definitions

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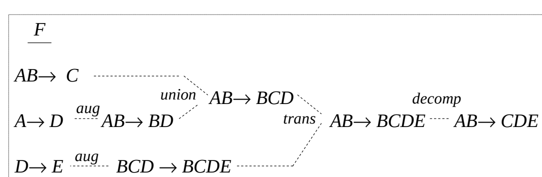
Generating F^+



Thus, $AB \rightarrow BD$, $AB \rightarrow BCD$, $AB \rightarrow BCDE$, and $AB \rightarrow CDE$ are all elements of F^+

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Attribute Closure

- Calculating *attribute closure* leads to a more efficient way of checking entailment
- The *attribute closure* of a set of attributes, X, with respect to a set of functional dependencies, F, (denoted X^+_F) is the set of all attributes, A, such that $X \rightarrow A$
  - X^+_{F1} is not necessarily the same as X^+_{F2} if $F1 \neq F2$
- *Attribute closure and entailment*:
  - Algorithm: Given a set of FDs, F, then $X \rightarrow Y$ if and only if $X^+_F \supseteq Y$

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Example - Computing Attribute Closure

F: $AB \rightarrow C$
$A \rightarrow D$
$D \rightarrow E$
$AC \rightarrow B$

X	X_F^+
A	$\{A, D, E\}$
AB	$\{A, B, C, D, E\}$ (Hence AB is a key)
B	$\{B\}$
D	$\{D, E\}$

Is $AB \rightarrow E$ entailed by F? *Yes*
Is $D \rightarrow C$ entailed by F? *No*

Result: X_F^+ allows us to determine FDs of the form $X \rightarrow Y$ entailed by F

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Computation of Attribute Closure X^+_F

closure := X; // *since* $X \subseteq X^+_F$
repeat
  old := *closure*;
  if there is an FD $Z \rightarrow V$ in F such that
    $Z \subseteq$ *closure* **and** $V \not\subseteq$ *closure*
   then *closure* := *closure* $\cup$ V
until *old* = *closure*

– If $T \subseteq$ *closure* then $X \rightarrow T$ is entailed by F

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Example: Computation of Attribute Closure

Problem: Compute the attribute closure of AB with respect to the set of FDs :
$AB \rightarrow C$ (a)
$A \rightarrow D$ (b)
$D \rightarrow E$ (c)
$AC \rightarrow B$ (d)

Solution:

Initially *closure* = $\{AB\}$
Using (a) *closure* = $\{ABC\}$
Using (b) *closure* = $\{ABCD\}$
Using (c) *closure* = $\{ABCDE\}$

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Normal Forms

- Each normal form is a set of conditions on a schema that guarantees certain properties (relating to redundancy and update anomalies)
- First normal form (1NF) is the same as the definition of relational model (relations = sets of tuples; each tuple = sequence of atomic values)
- Second normal form (2NF) – a research lab accident; has no practical or theoretical value – won't discuss
- The two commonly used normal forms are *third normal form* (3NF) and *Boyce-Codd normal form* (BCNF)

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BCNF

- **Definition**: A relation schema **R** is in BCNF if for every FD $X \rightarrow Y$ associated with **R** either
  - $Y \subseteq X$ (i.e., the FD is trivial) or
  - X is a superkey of **R**
- **Example**: Person1(*SSN*, *Name*, *Address*)
  - The only FD is $SSN \rightarrow Name, Address$
  - Since *SSN* is a key, Person1 is in BCNF

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(non) BCNF Examples

- Person (*SSN, Name, Address, Hobby*)
  - The FD *SSN* → *Name, Address* does not satisfy requirements of BCNF
    - since the key is (*SSN, Hobby*)
- HasAccount (*AccountNumber, ClientId, OfficeId*)
  - The FD *AcctNum*→ *OfficeId* does not satisfy BCNF requirements
    - since keys are (*ClientId, OfficeId*) and (*AcctNum, ClientId*)

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Redundancy

- Suppose **R** has a FD *A* → *B*. If an instance has 2 rows with same value in *A*, they *must* also have same value in *B* (=> redundancy, if the A-*value* repeats twice)

redundancy

SSN → *Name, Address*

SSN	*Name*	*Address*	*Hobby*
1111	Joe	123 Main	stamps
1111	Joe	123 Main	coins

- If *A* is a superkey, there cannot be two rows with same value of *A*
  - Hence, BCNF eliminates redundancy

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Third Normal Form

- A relational schema **R** is in 3NF if for every FD *X*→ *Y* associated with **R** either:
  - *Y* ⊆ *X* (i.e., the FD is trivial); or
  - *X* is a superkey of **R; ** or
  - Every *A*∈ *Y* is part of some key of **R**
- 3NF is weaker than BCNF (every schema that is in BCNF is also in 3NF)

BCNF conditions

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3NF Example

- HasAccount (*AcctNum, ClientId, OfficeId*)
  - *ClientId, OfficeId* → *AcctNum*
    - OK since LHS contains a key
  - *AcctNum* → *OfficeId*
    - OK since RHS is part of a key
- HasAccount is in 3NF but it might still contain redundant information due to *AcctNum* → *OfficeId* (which is not allowed by BCNF)

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3NF Example

- HasAccount might store redundant data:

ClientId	*OfficeId*	*AcctNum*
1111	Stony Brook	28315
2222	Stony Brook	28315
3333	Stony Brook	28315

3NF: *OfficeId* part of key
FD: *AcctNum* → *OfficeId*

redundancy

- Decompose to eliminate redundancy:

ClientId	*AcctNum*
1111	28315
2222	28315
3333	28315

BCNF (only trivial FDs)

OfficeId	*AcctNum*
Stony Brook	28315

BCNF: *AcctNum* is key
FD: *AcctNum* → *OfficeId*

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3NF (Non) Example

- Person (*SSN, Name, Address, Hobby*)
  - (*SSN, Hobby*) is the only key.
  - *SSN*→*Name* violates 3NF conditions since *Name* is not part of a key and *SSN* is not a superkey

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Decompositions

- **Goal**: Eliminate redundancy by decomposing a relation into several relations in a higher normal form
- Decomposition must be *lossless*: it must be possible to reconstruct the original relation from the relations in the decomposition
  - We will see why

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Decomposition

- Schema $\mathbf{R} = (R, F)$
  - R is set a of attributes
  - F is a set of functional dependencies over R
    - Each key is described by a FD
- The *decomposition of schema* **R** is a collection of schemas $\mathbf{R}_i = (R_i, F_i)$ where
  - $R = \cup_i R_i$ for all i (*no new attributes*)
  - F_i is a set of functional dependences involving only attributes of R_i
  - F entails F_i for all i (*no new FDs*)
- The *decomposition of an instance*, **r**, of **R** is a set of relations $\mathbf{r}_i = \pi_{R_i}(\mathbf{r})$ for all i

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Example Decomposition

Schema (R, F) where
$R = \{SSN, Name, Address, Hobby\}$
$F = \{SSN \rightarrow Name, Address\}$
can be decomposed into
$R_1 = \{SSN, Name, Address\}$
$F_1 = \{SSN \rightarrow Name, Address\}$
and
$R_2 = \{SSN, Hobby\}$
$F_2 = \{ \}$

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Lossless Schema Decomposition

- A decomposition should not lose information
- A decomposition $(\mathbf{R}_1, \ldots, \mathbf{R}_n)$ of a schema, **R**, is *lossless* if every valid instance, **r**, of **R** can be reconstructed from its components:

$$\mathbf{r} = \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots\ldots \bowtie \mathbf{r}_n$$

- where each $\mathbf{r}_i = \pi_{\mathbf{R}_i}(\mathbf{r})$

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Lossy Decomposition

The following is always the case (Think why?):

$$\mathbf{r} \subseteq \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots \bowtie \mathbf{r}_n$$

But the following is not always true:

$$\mathbf{r} \supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots \bowtie \mathbf{r}_n$$

Example: $\mathbf{r} \not\supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2$

SSN	*Name*	*Address*
1111	Joe	1 Pine
2222	Alice	2 Oak
3333	Alice	3 Pine

SSN	*Name*
1111	Joe
2222	Alice
3333	Alice

Name	*Address*
Joe	1 Pine
Alice	2 Oak
Alice	3 Pine

The tuples (2222, Alice, 3 Pine) and (3333, Alice, 2 Oak) *are in the join, but not in the original*

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Lossy Decompositions: *What is Actually Lost*?

- In the previous example, the tuples (2222, Alice, 3 Pine) and (3333, Alice, 2 Oak) were *gained*, not lost!
  - Why do we say that the decomposition was lossy?
- What was lost is *information*:
  - That 2222 lives at 2 Oak: *In the decomposition, 2222 can live at either 2 Oak or 3 Pine*
  - That 3333 lives at 3 Pine: *In the decomposition, 3333 can live at either 2 Oak or 3 Pine*

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Testing for Losslessness

- A (binary) decomposition of **R** = (R, F) into $\mathbf{R}_1 = (R_1, F_1)$ and $\mathbf{R}_2 = (R_2, F_2)$ is lossless *if and only if* :
  - either the FD
    - $(R_1 \cap R_2) \rightarrow R_1$ is in F^+
  - or the FD
    - $(R_1 \cap R_2) \rightarrow R_2$ is in F^+

Example

Schema (R, F) where

$R = \{SSN, Name, Address, Hobby\}$

$F = \{SSN \rightarrow Name, Address\}$

can be decomposed into

$R_1 = \{SSN, Name, Address\}$

$F_1 = \{SSN \rightarrow Name, Address\}$

and

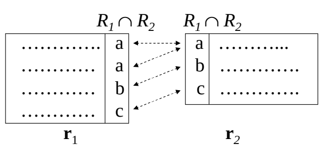
$R_2 = \{SSN, Hobby\}$

$F_2 = \{ \}$

Since $R_1 \cap R_2 = SSN$ and $SSN \rightarrow R_1$ the decomposition is lossless

Intuition Behind the Test for Losslessness

- Suppose $R_1 \cap R_2 \rightarrow R_2$. Then a row of $\mathbf{r}_1$ can combine with exactly one row of $\mathbf{r}_2$ in the natural join (since in $\mathbf{r}_2$ a particular set of values for the attributes in $R_1 \cap R_2$ defines a unique row)



Proof of Lossless Condition

- $\mathbf{r} \subseteq \mathbf{r}_1 \bowtie \mathbf{r}_2$ – *this is true for any decomposition*
- $\mathbf{r} \supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2$

  If $R_1 \cap R_2 \rightarrow R_2$ then

  $card\ (\mathbf{r}_1 \bowtie \mathrm{r}_2) = card\ (\mathbf{r}_1)$

  (since each row of r_1 joins with exactly one row of r_2)

  But $card\ (\mathbf{r}) \geq card\ (\mathbf{r}_1)$ (*since* $\mathbf{r}_1$ *is a projection of* $\mathbf{r}$) and therefore $card\ (\mathbf{r}) \geq card\ (\mathbf{r}_1 \bowtie \mathbf{r}_2)$

  Hence $\mathbf{r} = \mathbf{r}_1 \bowtie \mathbf{r}_2$

Dependency Preservation

- Consider a decomposition of **R** = (R, F) into $\mathbf{R}_1 = (R_1, F_1)$ and $\mathbf{R}_2 = (R_2, F_2)$
  - An FD $X \rightarrow Y$ of F is in F_i iff $X \cup Y \subseteq R_i$
  - An FD, $f \in F$ may be in neither F_1, nor F_2, nor even $(F_1 \cup F_2)^+$
    - Checking that f is true in $\mathbf{r}_1$ or $\mathbf{r}_2$ is (relatively) easy
    - Checking f in $\mathbf{r}_1 \bowtie \mathbf{r}_2$ is harder – requires a join
    - *Ideally*: want to check FDs <u>locally</u>, in $\mathbf{r}_1$ and $\mathbf{r}_2$, and have a guarantee that every $f \in F$ holds in $\mathbf{r}_1 \bowtie \mathbf{r}_2$
- The decomposition is *dependency preserving* iff the sets F and $F_1 \cup F_2$ are equivalent: $F^+ = (F_1 \cup F_2)^+$
  - Then checking all FDs in F, as $\mathbf{r}_1$ and $\mathbf{r}_2$ are updated, can be done by checking F_1 in $\mathbf{r}_1$ and F_2 in $\mathbf{r}_2$

Dependency Preservation

- If f is an FD in F, but f is not in $F_1 \cup F_2$, there are two possibilities:
  - $f \in (F_1 \cup F_2)^+$
    - If the constraints in F_1 and F_2 are maintained, f will be maintained automatically.
  - $f \notin (F_1 \cup F_2)^+$
    - f can be checked only by first taking the join of r_1 and r_2. This is costly.

Example

Schema (R, F) where

R = {SSN, $Name$, $Address$, $Hobby$}

F = {$SSN \rightarrow Name, Address$}

can be decomposed into

R_1 = {SSN, $Name$, $Address$}

F_1 = {$SSN \rightarrow Name, Address$}

and

R_2 = {SSN, $Hobby$}

F_2 = { }

Since $F = F_1 \cup F_2$ the decomposition is dependency preserving

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Example

- Schema: (ABC; F) , $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow B\}$
- Decomposition:
  - (AC, F_1), $F_1 = \{A \rightarrow C\}$
    - Note: $A \rightarrow C \notin F$, but in F^+
  - (BC, F_2), $F_2 = \{B \rightarrow C, C \rightarrow B\}$
- $A \rightarrow B \notin (F_1 \cup F_2)$, **but** $A \rightarrow B \in (F_1 \cup F_2)^+$.
  - So $F^+ = (F_1 \cup F_2)^+$ and thus the decompositions is still dependency preserving

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Example

- HasAccount (*AccountNumber*, *ClientId*, *OfficeId*)
  - f_1: *AccountNumber* → *OfficeId*
  - f_2: *ClientId*, *OfficeId* → *AccountNumber*
- Decomposition:
  - AcctOffice = (*AccountNumber*, *OfficeId*; {*AccountNumber* → *OfficeId*})
  - AcctClient = (*AccountNumber*, *ClientId*; {})
- Decomposition is lossless: $R_1 \cap R_2$= {*AccountNumber*} and *AccountNumber* → *OfficeId*
- In BCNF
- Not dependency preserving: $f_2 \notin (F_1 \cup F_2)^+$
- HasAccount *does not* have BCNF decompositions that are both lossless and dependency preserving! (Check, eg, by enumeration)
- Hence: BCNF+lossless+dependency preserving decompositions are not always achievable!

BCNF Decomposition Algorithm

***Input*:** **R** = (R; F)

Decomp := **R**

while there is **S** = (S; F') ∈ *Decomp* and **S** not in BCNF **do**

Find $X \rightarrow Y \in F'$ that violates BCNF // *X isn't a superkey in* **S**

Replace **S** in *Decomp* with $\mathbf{S_1} = (XY; F_1)$, $\mathbf{S_2} = (S - (Y - X); F_2)$

// F_1 = *all FDs of F' involving only attributes of XY*

// F_2 = *all FDs of F' involving only attributes of S - (Y - X)*

end

return *Decomp*

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Example

Given: **R** = (R; T) where R = *ABCDEFGH* and

$T = \{ABH \rightarrow C, A \rightarrow DE, BGH \rightarrow F, F \rightarrow ADH, BH \rightarrow GE\}$

step 1: Find a FD that violates BCNF

Not $ABH \rightarrow C$ since $(ABH)^+$ includes all attributes (BH is a key)

$A \rightarrow DE$ violates BCNF since A is not a superkey ($A^+ = ADE$)

step 2: Split **R** into:

$\mathbf{R_1}$ = (ADE, $\{A \rightarrow DE\}$)

$\mathbf{R_2}$ = ($ABCFGH$; $\{ABH \rightarrow C, BGH \rightarrow F, F \rightarrow AH, BH \rightarrow G\}$)

Note 1: $\mathbf{R_1}$ is in BCNF

Note 2: Decomposition is *lossless* since A is a key of $\mathbf{R_1}$.

Note 3: FDs $F \rightarrow D$ and $BH \rightarrow E$ are not in T_1 or T_2. But both can be derived from $T_1 \cup T_2$

(*E.g.*, $F \rightarrow A$ and $A \rightarrow D$ implies $F \rightarrow D$)

Hence, decomposition is *dependency preserving*.

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Properties of BCNF Decomposition Algorithm

Let $X \rightarrow Y$ violate BCNF in **R** = (R,F) and $\mathbf{R_1} = (R_1, F_1)$, $\mathbf{R_2} = (R_2, F_2)$ is the resulting decomposition. Then:

- There are *fewer violations* of BCNF in $\mathbf{R_1}$ and $\mathbf{R_2}$ than there were in **R**
  - $X \rightarrow Y$ implies X is a key of $\mathbf{R_1}$
  - Hence $X \rightarrow Y \in F_1$ does not violate BCNF in $\mathbf{R_1}$ and, since $X \rightarrow Y \notin F_2$, does not violate BCNF in $\mathbf{R_2}$ either
  - Suppose f is $X' \rightarrow Y'$ and $f \in F$ doesn't violate BCNF in **R**. **If** $f \in F_1$ or F_2 it does not violate BCNF in $\mathbf{R_1}$ or $\mathbf{R_2}$ either since X' is a superkey of **R** and hence also of $\mathbf{R_1}$ and $\mathbf{R_2}$.
- The decomposition is *lossless*
  - Since $F_1 \cap F_2 = X$

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Example (con't)

Given: $\mathbf{R_2}$ **=** $(ABCFGH; \{ABH \rightarrow C, BGH \rightarrow F, F \rightarrow AH, BH \rightarrow G\})$
step 1: Find a FD that violates BCNF.
Not $ABH \rightarrow C$ or $BGH \rightarrow F$, since BH is a key of $\mathbf{R_2}$
$F \rightarrow AH$ violates BCNF since F is not a superkey ($F^+ = AH$)
step 2: Split $\mathbf{R_2}$ into:
$\mathbf{R_{21}}$ = $(FAH, \{F \rightarrow AH\})$
$\mathbf{R_{22}}$ = $(BCFG; \{\})$

Note 1: Both $\mathbf{R_{21}}$ and $\mathbf{R_{22}}$ are in BCNF.
Note 2: The decomposition is *lossless* (since F is a key of $\mathbf{R_{21}}$)
Note 3: FDs $ABH \rightarrow C$, $BGH \rightarrow F$, $BH \rightarrow G$ are not in T_{21} or T_{22}, and they can't be derived from $T_1 \cup T_{21} \cup T_{22}$.
Hence the decomposition is *not* dependency-preserving

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Properties of BCNF Decomposition Algorithm

- A BCNF decomposition is *not necessarily* dependency preserving
- But *always* lossless
- BCNF+lossless+dependency preserving is sometimes unachievable (recall HasAccount)

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Third Normal Form

- Compromise – Not all redundancy removed, but dependency preserving decompositions are <u>always</u> possible (and, of course, lossless)
- 3NF decomposition is based on a *minimal cover*

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Minimal Cover

- A *minimal cover* of a set of dependencies, T, is a set of dependencies, U, such that:
  - U is equivalent to T $(T^+ = U^+)$
  - All FDs in U have the form $X \rightarrow A$ where A is a single attribute
  - It is not possible to make U smaller (while preserving equivalence) by
    - Deleting an FD
    - Deleting an attribute from an FD (either from LHS or RHS)
  - FDs and attributes that can be deleted in this way are called *redundant*

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Computing Minimal Cover

- **Example**: $T = \{ABH \rightarrow CK, A \rightarrow D, C \rightarrow E, BGH \rightarrow F, F \rightarrow AD, E \rightarrow F, BH \rightarrow E\}$
- **step 1**: Make RHS of each FD into a single attribute
  - *Algorithm*: Use the decomposition inference rule for FDs
  - Example: $F \rightarrow AD$ replaced by $F \rightarrow A, F \rightarrow D$; $ABH \rightarrow CK$ by $ABH \rightarrow C$, $ABH \rightarrow K$
- **step 2**: Eliminate redundant attributes from LHS.
  - *Algorithm*: If FD $XB \rightarrow A \in T$ (where B is a single attribute) and $X \rightarrow A$ is entailed by T, then B was unnecessary
  - Example: Can an attribute be deleted from $ABH \rightarrow C$?
    - Compute AB^+_T, AH^+_T, BH^+_T.
    - Since $C \in (BH)^+_T$, $BH \rightarrow C$ is entailed by T and A is redundant in $ABH \rightarrow C$.

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Computing Minimal Cover (con't)

- **step 3**: Delete redundant FDs from T
  - *Algorithm*: If $T - \{f\}$ entails f, then f is redundant
    - If f is $X \rightarrow A$ then check if $A \in X^+_{T-\{f\}}$
  - Example: $BGH \rightarrow F$ is entailed by $E \rightarrow F$, $BH \rightarrow E$, so it is redundant
- *Note*: The order of steps 2 and 3 cannot be interchanged!! See the textbook for a counterexample

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Synthesizing a 3NF Schema

Starting with a schema $\mathbf{R} = (R, T)$

- **step 1**: Compute a minimal cover, U, of T. The decomposition is based on U, but since $U^+ = T^+$ the same functional dependencies will hold
  - A minimal cover for $T=\{ABH \rightarrow CK, A \rightarrow D, C \rightarrow E, BGH \rightarrow F, F \rightarrow AD, E \rightarrow F, BH \rightarrow E\}$

    is

    $U=\{BH \rightarrow C, BH \rightarrow K, A \rightarrow D, C \rightarrow E, F \rightarrow A, E \rightarrow F\}$

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Synthesizing a 3NF schema (con't)

- **step 2**: Partition U into sets $U_1, U_2, \ldots U_n$ such that the LHS of all elements of U_i are the same
  - $U_1 = \{BH \rightarrow C, BH \rightarrow K\}$, $U_2 = \{A \rightarrow D\}$, $U_3 = \{C \rightarrow E\}$, $U_4 = \{F \rightarrow A\}$, $U_5 = \{E \rightarrow F\}$

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Synthesizing a 3NF schema (con't)

- **step 3**: For each U_i form schema $\mathbf{R_i} = (R_i, U_i)$, where R_i is the set of all attributes mentioned in U_i
  - Each FD of U will be in some $\mathbf{R_i}$. Hence the decomposition is *dependency preserving*
  - $\mathbf{R_1} = (BHC;\ BH \rightarrow C, BH \rightarrow K)$, $\mathbf{R_2} = (AD;\ A \rightarrow D)$, $\mathbf{R_3} = (CE;\ C \rightarrow E)$, $\mathbf{R_4} = (FA;\ F \rightarrow A)$, $\mathbf{R_5} = (EF;\ E \rightarrow F)$

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Synthesizing a 3NF schema (con't)

- **step 4**: If no R_i is a superkey of **R**, add schema $\mathbf{R_0} = (R_0, \{\})$ where R_0 is a key of **R**.
  - $\mathbf{R_0} = (BGH, \{\})$
    - $\mathbf{R_0}$ might be needed when not all attributes are necessarily contained in $R_1 \cup R_2 \ldots \cup R_n$
      - A missing attribute, A, must be part of all keys

        (since it's not in any FD of U, deriving a key constraint from U involves the augmentation axiom)
    - $\mathbf{R_0}$ might be needed even if all attributes are accounted for in $R_1 \cup R_2 \ldots \cup R_n$
      - Example: $(ABCD; \{A \rightarrow B, C \rightarrow D\})$. Step 3 decomposition: $R_1 = (AB; \{A \rightarrow B\})$, $R_2 = (CD; \{C \rightarrow D\})$. Lossy! Need to add (AC; { }), for losslessness
  - Step 4 guarantees lossless decomposition.

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BCNF Design Strategy

- The resulting decomposition, $\mathbf{R_0}, \mathbf{R_1}, \ldots \mathbf{R_n}$, is
  - Dependency preserving (since every FD in U is a FD of some schema)
  - Lossless (although this is not obvious)
  - In 3NF (although this is not obvious)
- Strategy for decomposing a relation
  - Use 3NF decomposition first to get lossless, dependency preserving decomposition
  - If any resulting schema is not in BCNF, split it using the BCNF algorithm (but this may yield a non-dependency preserving result)

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Normalization Drawbacks

- By limiting redundancy, normalization helps maintain consistency and saves space
- But performance of querying can suffer because related information that was stored in a single relation is now distributed among several
- **Example**: A join is required to get the names and grades of all students taking CS305 in S2002.

```
SELECT  S.Name, T.Grade
FROM  Student S, Transcript T
WHERE  S.Id = T.StudId  AND
        T.CrsCode = 'CS305'  AND  T.Semester = 'S2002'
```

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Denormalization

- **Tradeoff**: *Judiciously* introduce redundancy to improve performance of certain queries
- **Example**: Add attribute *Name* to Transcript

```
SELECT  T.Name, T.Grade
FROM    Transcript′ T
WHERE   T.CrsCode = 'CS305' AND T.Semester = 'S2002'
```

  – Join is avoided
  – If queries are asked more frequently than Transcript is modified, added redundancy might improve average performance
  – But, Transcript′ is no longer in BCNF since key is (*StudId, CrsCode, Semester*) and *StudId* $\rightarrow$ *Name*

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Fourth Normal Form

SSN	*PhoneN*	*ChildSSN*
111111	123-4444	222222
111111	123-4444	333333
222222	987-6666	444444
222222	555-5555	444444

redundancy

Person

- Relation has redundant data
- Yet it is in BCNF (since there are no non-trivial FDs)
- Redundancy is due to set valued attributes (in the E-R sense), not because of the FDs

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Multi-Valued Dependency

- **Problem**: multi-valued (or binary join) dependency
  – **Definition**: If every instance of schema **R** can be (losslessly) decomposed using attribute sets (X, Y) such that:

$$\mathbf{r} = \pi_X(\mathbf{r}) \bowtie \pi_Y(\mathbf{r})$$

then a *multi-valued dependency*

$$\mathbf{R} = \pi_X(\mathbf{R}) \bowtie \pi_Y(\mathbf{R})$$

holds in **r**

Ex: Person $= \pi_{SSN,PhoneN}$(Person) $\bowtie$ $\pi_{SSN,ChildSSN}$(Person)

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Fourth Normal Form (4NF)

- A schema is in *fourth normal form* (4NF) if for every non-trivial multi-valued dependency:

$$R = X \bowtie Y$$

either:

- $X \subseteq Y$ or $Y \subseteq X$ (trivial case); or
- $X \cap Y$ is a superkey of R (*i.e.*, $X \cap Y \rightarrow R$)

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Fourth Normal Form (Cont'd)

- *Intuition*: if $X \cap Y \rightarrow R$, there is a unique row in relation **r** for each value of $X \cap Y$ (hence no redundancy)
  – Ex: *SSN* does not uniquely determine *PhoneN* or *ChildSSN*, thus Person is not in 4NF.
- *Solution*: Decompose R into X and Y
  – Decomposition is lossless – but not necessarily dependency preserving (since 4NF implies BCNF – next)

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4NF Implies BCNF

- Suppose R is in 4NF and $X \rightarrow Y$ is an FD.
  – $R1 = XY$, $R2 = R - Y$ is a lossless decomposition of R
  – Thus R has the multi-valued dependency:

$$R = R_1 \bowtie R_2$$

  – Since R is in 4NF, one of the following must hold :
    – $XY \subseteq R - Y$ (an impossibility)
    – $R - Y \subseteq XY$ (i.e., $R = XY$ and X is a superkey)
    – $XY \cap R - Y$ ($= X$) is a superkey
  – Hence $X \rightarrow Y$ satisfies BCNF condition

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