1. (9 points) Jim, a realtor for Century 3000, decides to computerize his office work. He decides that it is best if he organizes everything into a database application. Please help him in organizing the information about his properties including houses, apartments, and lands that he owns into an E/R diagram. He said that he needs the following information (Jim is not a computer scientist and his information might not be well organized):

- For each house, he would like to store the information such as the address (street, number, zip code), the list of owners (this goes as far as he has the information), and the monthly payment that he would receive. Since he sometimes needs to contact the owner of the house, he often keeps the name, the social security number, and the phone number of the owners. It is also customary that he might have some houses that he just bought and does not sell it back to anyone yet. He calls them as pending houses. Each pending house is often assigned a sale deadline (the latest day the house should be put on the market for sale).

- The information that he wants to store about the apartments includes the address (street, number, zip code), the current tenant (name, ssn, phone number), and the monthly rental fee.

- Regarding the lands, he needs its location (street, number, zip code), its value, the date of purchase, and whether it is a residential or commercial land.

- For the houses and the apartments that he is collecting monthly payments, he needs to know if a buyer (or a tenant) does not make the payments for three months then he needs to dismiss the contract between him and the buyer (or the tenant). For this reason, he records the payment of his customers into two books, one for the houses and one for the apartments.

Assume that

- For each property, Jim had assigned a unique code when he bought it (for example, H001, H002, … etc. for the houses, A001, A002, … etc. for the apartments, and L001, L002, … etc. for the lands)

- No one owns a house and rents an apartment at the same time.
- Payment must be made on time and in the exact amount as specified and agreed by his customers.

Create a E/R diagram for Jim’s properties. For each entity set in your E/R diagram, specify a key. Specify the multiplicity of relationships in your diagram clearly. If you think that you need some additional assumptions, state them clearly and work with it.
2. (4 points) Translate the following E/R diagram into a relational database schema: (Give the keys for the relations obtained from the translation): (Test 1A)

![E/R diagram]

Star (Name, Zipcode, Phone, Street_and_No)
Movie (Name, Year, Length, Type)
Movie_Studio (Name, Est_Value, NoOfEmployees, YearFounded, President, Address)
Movie_Contract (StarName, MovieName, Year, StudioOfStar, ProducingStudio)
3. (7 points) Suppose that we have a relation $R(\text{A,B,C,D,E})$ with the following functional dependencies: (Test 1A)

\[ \text{AB} \rightarrow \text{DE}, \text{C} \rightarrow \text{E}, \text{D} \rightarrow \text{C}, \text{and} \text{E} \rightarrow \text{A} \]

3.1 Is $R$ in BCNF form? If not, list ALL the functional dependencies that violate the BCNF condition and state the reasons why they violate the BCNF. (Compute the closure of the set of attributes on the left hand side of each functional dependency is a good idea. Do it step by step!)

Computing: $\{\text{A,B}\}^+$

$X = \{\text{A,B}\}$, $\text{AB} \rightarrow \text{DE}$ has the left hand side in $X$

$\Rightarrow X = \{\text{A,B}\} \cup \{\text{D,E}\}, \text{D} \rightarrow \text{C}$ has the left hand side in $X$

$\Rightarrow X = \{\text{A,B,D,E}\} \cup \{\text{C}\} = \{\text{A,B,D,E,C}\}$

$\Rightarrow \{\text{A,B}\}^+ = \{\text{A, B, C, D, E}\}$

$\Rightarrow \{\text{A,B}\}$ is a superkey, $\text{AB} \rightarrow \text{DE}$ does not violates the BCNF condition

Similarly:

$\{\text{C}\}^+ = \{\text{C, E, A}\} \Rightarrow \{\text{C}\}$ is not a key, $\text{C} \rightarrow \text{E}$ violates the BCNF condition

$\{\text{D}\}^+ = \{\text{D, C, E, A}\} \Rightarrow \{\text{D}\}$ is not a key, $\text{D} \rightarrow \text{C}$ violates the BCNF

$\{\text{E}\}^+ = \{\text{E, A}\} \Rightarrow \{\text{E}\}$ is not a key, $\text{E} \rightarrow \text{A}$ violates the BCNF

Thus, $R$ is not in BCNF form.

3.2 Suppose that we decompose the relation $R$ into two relations $S(\text{A,B,C})$ and $T(\text{A,B,D,E})$. List all the functional dependencies that hold in $S$. Justify your answer with necessary computation.

Because of the above computation:

$\{\text{A,B}\}^+ = \{\text{A, B, C, D, E}\} \Rightarrow \text{we have AB} \rightarrow \text{C holds in S}$

$\{\text{C}\}^+ = \{\text{C, E, A}\} \Rightarrow \text{C} \rightarrow \text{A}$ holds in S
4. (4 points) Translate the following E/R diagram into a relational database schema:
(Give the keys for the relations obtained from the translation): (Test 1B)

Movie_Star (Name, Zipcode, Phone, Street_and_No)
Movie (Name, Year, Length, Type)
Studio (Name, Est_Value, NoOfEmployees, YearFounded, President, Address)
Coop_Contract (MovieStarName, MovieName, Year, StudioOfStar, ProducingStudio)
5. (7 points) Suppose that we have a relation \( R(A,B,C,D,E) \) with the following functional dependencies: (Test 1B)

\[ AC \rightarrow DE, \ B \rightarrow D, \ E \rightarrow B, \text{ and } D \rightarrow A \]

3.1 Is \( R \) in BCNF form? If not, list ALL the functional dependencies that violate the BCNF condition and state the reasons why they violate the BCNF. (Compute the closure of the set of attributes on the left hand side of each functional dependency is a good idea. Do it step by step!)

Computing: \( \{A,C\}^+ \)

\[ X = \{A,C\} \cup \{D,E\}, \ E \rightarrow B \text{ has the left hand side in } X \]
\[ \Rightarrow X=\{A,C,D,E\} \cup \{B\} = \{A,B,D,E,C\} \]
\[ \Rightarrow \{A,C\}^+ = \{A, B, C, D, E\} \]
\[ \Rightarrow \{A,C\} \text{ is a superkey. } AC \rightarrow DE \text{ does not violates the BCNF condition} \]

Similarly:

\( \{B\}^+ = \{B, D, A\} \Rightarrow \{B\} \text{ is not a key, } B \rightarrow D \text{ violates the BCNF condition} \)
\( \{E\}^+ = \{E, B, D, A\} \Rightarrow \{E\} \text{ is not a key, } E \rightarrow B \text{ violates the BCNF condition} \)
\( \{D\}^+ = \{D, A\} \Rightarrow \{D\} \text{ is not a key, } D \rightarrow A \text{ violates the BCNF condition} \)

3.2 Suppose that we decompose the relation \( R \) into two relations \( S(A,B,C) \) and \( T(A,B,D,E) \). List all the functional dependencies that hold in \( S \). Justify your answer with necessary computation.

Because of the above computation:

\( \{A,C\}^+ = \{A, B, C, D, E\} \Rightarrow \text{ we have } AC \rightarrow B \text{ holds in } S \)
\( \{B\}^+ = \{B, D, A\} \Rightarrow B \rightarrow A \text{ holds in } S \)
6. (2 points) Let $S$ and $T$ be two sets of attributes of a relation $R$. Prove that if $S \subseteq T$ then $S^+ \subseteq T^+$.

Let $F$ be the set of FDs that hold in $R$.
It is easy to see that the following algorithm can be used to compute $S^+$:

Step 1: Set $X = S$
Step 2: Let $Z = \{ B \mid$ there exists a $A_1 A_2, \ldots, A_m \rightarrow B$ in $F$
and $\{ A_1, A_2, \ldots, A_m \} \subseteq X \}$
Step 3: if $Z \setminus X \neq \emptyset$ then set $X = X \cup Z$ and repeat step 2.
Otherwise, stop and return $X$ (which is equals $X \cup Z$ ) as $S^+$.

Because of the finiteness of the set of attributes of $R$, we can conclude that the above algorithm will stop and it generates a sequence of sets of attributes

$$S = X_1 \subset X_2 \subset \ldots \subset X_p = S^+.$$  

Similarly, we can generate $T^+$ by the above algorithm and the sequence of sets of attributes that generates $T^+$ is $T = Y_1 \subset Y_2 \subset \ldots \subset Y_q = T^+$.

We will consider two sequences of sets of attributes:

$X_1, X_2, \ldots, X_p = X_{p+1}, \ldots$ and $X_1, Y_2, \ldots, Y_q = Y_{q+1}, \ldots$.

We prove that $X_k \subseteq Y_k$ for every integer $k$ by induction over $k$:

Base case: By the assumption $S \subseteq T$.

Inductive case: We need to prove that if $X_k \subseteq Y_k$ then $X_{k+1} \subseteq Y_{k+1}$.

Let $Z_1$ and $Z_2$ be the set $Z$ obtained in the step 2 of the above algorithm for $X_k$ and $Y_k$, respectively. Obviously $Z_1 \subseteq Z_2$. This implies that $X_{k+1} = X_k \cup Z_1 \subseteq Y_{k+1} = Y_k \cup Z_2$.

The inductive case is proved. So, we have proved that $X_k \subseteq Y_k$ for every integer $k$.

So, for an integer $m$ such that $m > p$ and $m > q$, we have that

$$S^+ = X_m \subseteq Y_m = T^+.$$