EXAMPLES OF RELATIONS

Relations can be completely arbitrary in which objects are chosen to be related – some elements in either set can be omitted from the relation, and objects can participate in more than one pairing, either way:

We could write this relation as \( A \rho B \). The Cartesian product, written as \( A \times B \) connects every object in one set to every object in the other set. Clearly, then, \( A \rho B \subseteq A \times B \).

A simple example is the relation ‘greater-than’ between natural numbers (i.e. the set \( \{0, 1, 2, 3, \ldots \} \)). This could be written as: \( \{[x, y] | \forall x, y \in \mathbb{N}, x > y \} \) or, equivalently as \( \forall [x, y] \in \mathbb{N} \times \mathbb{N}, x \text{ greaterThan } y \Leftrightarrow x > y \). (\( \mathbb{N} \) is a common way to write the set of all natural numbers.) The relation set starts out: \( \{[[1, 0], [2, 1], [2, 0], [3, 2], [3, 1], [3, 0], \ldots]\} \). Notice that the pair is ordered: e.g. \([0, 1]\) is not in the relation.

A more complex example is the relation subset on a set of sets \( A \): \( \forall S, T \in A, S \subseteq T \Leftrightarrow \forall x, x \in S \Rightarrow x \in T \). If \( A \) is \( \{\{a, b, c\}, \{a, b\}, \{a\}\} \) then \( A \) is such a set, because \( \{a\} \subseteq \{a, b\} \subseteq \{a, b, c\} \). Notice, for instance, it is not the case that \( \{a, b\} \subseteq \{a\} \) since it does not satisfy the condition on the membership of each set. On the other hand, the set \( \{\{a\}, \{b, c\}, \{d\}\} \) does not follow the relation since no set is a subset of any other.