EXAMPLES OF FUNCTIONS

There are numerous ways to write the square function:

1. As an extensional (infinite) set of pairs: \( \{(1.2, 1.44), (3.7, 13.69), (0.0, 0.0), \ldots\} \)

2. As an intensional set of pairs: \( \{(x, y) \mid \forall x \in \mathbb{R}, y = x^2\} \)

3. As a relation: \( \forall [x, y] \in \mathbb{R} \times \mathbb{R}, x \text{ square } y \iff y = x^2 \)

4. As a typed lambda expression: \( \lambda x. x^2 \)

5. Functionality: \( \text{square} : \mathbb{R} \rightarrow \mathbb{R} \)

6. Formula: \( \text{square} : x \rightarrow x^2 \)

7. “standard” math form: \( \text{square}(x) = x^2 \), for all \( x \)

If we have the functions \( \text{square} : x \rightarrow x^2 \) and \( \text{successor} : x \rightarrow x + 1 \), then the composition \( \text{square} \circ \text{successor} \) is the function \( \text{squareSuccessor} : x \rightarrow (x + 1)^2 \), since \( \text{successor}(x) \) is \( x + 1 \) and \( \text{square}(x + 1) \) is \( (x + 1)^2 \).

Functions with more than one argument can be written as, e.g.:

1. \( \lambda x, y. x + 2y \)

2. \( f(x, y) = x + 2y \)

3. \( \{(x, y, z) \mid \forall z, y, z \in \mathbb{R}, z = x + 2y\} \)

although it can be shown that a function of more than one argument can be “curried” (after mathematician Haskell Curry) into a composition of single argument functions:

\( \lambda x. \lambda y. x + 2y \) is the same as \( \lambda x. \lambda y. x + 2y \).

The application of a function can be achieved by textual substitution of the argument into the body of the function. Whether to evaluate the argument first is a matter of choice, but the most common way is to completely evaluate the argument before substitution. E.g.

(standard notation) \( \text{square}(\text{square}(3 + 1)) = \text{square}(\text{square}(4)) = \text{square}(16) = 256 \)

(lambd notation) \( (\lambda x. x^2)(\lambda x. x^2(1 + 3)) = (\lambda x. x^2)(\lambda x. x^2(4)) = \lambda x. x^2(16) = 256 \)

[Notice how the lambda form has everything right there, including the body of the function, whereas the standard form has to “remember” the definition body.]

An updated function can also be applied: \( (\lambda x \mapsto 1)\{\} \) is undefined, since the function does not map 3 to anything.