

Four operations on the list: car, cdr, null?, cons

$(\text{car } '(a\ b\ c)) \Rightarrow a$

$(\text{cdr } '(a\ b\ c)) \Rightarrow (b\ c)$

Neither car nor cdr change the list they operate on.

e.g.  $(\text{define } x\ '(a\ b\ c))$

$x$   
 $\Rightarrow (a\ b\ c)$

$(\text{car } x)$

$\Rightarrow a$

$x$   
 $\Rightarrow (a\ b\ c)$

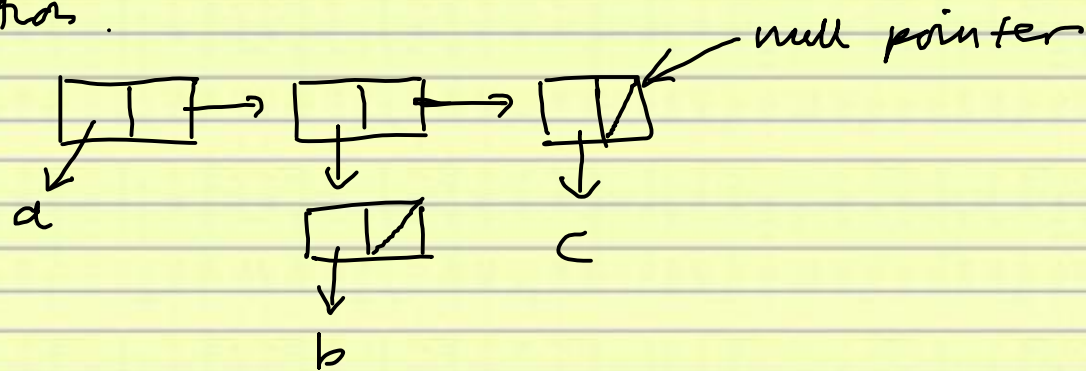
$(\text{cdr } x)$

$\Rightarrow (b\ c)$

$x$   
 $\Rightarrow (a\ b\ c)$

This is an aspect of functional languages called "referential transparency". The value returned by a function is always the same, given the same inputs. Side effects are not possible because there are no variables.

We can represent a list using box-and-arrow notation.



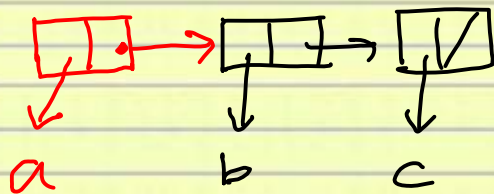
is the list (a (b) c)

Next operation: cons (construct)

cons builds new lists

$(\text{cons } 'a \ '(b \ c)) \Rightarrow (a \ b \ c)$

$(\text{cons } '(1) \ '(2 \ 3)) \Rightarrow ((1) \ 2 \ 3)$



The old list remains unchanged:

$(\text{define } x \ '(b \ c))$

$(\text{cons } 'a \ '(b \ c))$

$\Rightarrow (a \ b \ c)$

$x$

$\Rightarrow (b \ c)$

Last operation: `null?` - tests whether its argument is the empty list.

`(null? '()) => #t`

`(null? '(a b c)) => #f`

`(null? 1) => #f`

`(null? 'a) => #f`

We can drop the quote on the empty list:

`(null? ()) => #t`

## Simple functions for list processing (LISP)

## 1. length

e.g.  $(\text{length } '(a\ b\ c)) \Rightarrow 3$  $(\text{length } ()) \Rightarrow \emptyset$  $(\text{length } '((a\ b)\ c)) \Rightarrow 2$ 

(define length

(lambda (L)

(if (null? L)

 $\emptyset$ 

(+ 1 (length (cdr L))))))

Sample derivation:

$$\begin{aligned}
 & (\text{length } '(a\ b\ c)) \\
 = & (\text{if } (\text{null? } '(a\ b\ c)) \\
 & \quad \emptyset \\
 & \quad (+\ 1\ (\text{length } (\text{cdr } '(a\ b\ c)))))) \\
 = & (+\ 1\ (\text{length } '(b\ c))) \\
 = & (+\ 1\ (+\ 1\ (\text{length } '(c)))) \\
 = & (+\ 1\ (+\ 1\ (+\ 1\ (\text{length } ()))))) \\
 = & (+\ 1\ (+\ 1\ (+\ 1\ \emptyset))) \\
 = & 3
 \end{aligned}$$

The additions are suspended until the recursion "unwinds".

2. member? - treats the list as a set

e.g. (member? 'b '(a b c)) => #t

(member? 'b '(1 2 3)) => #f

We need an operation to test for equality of atoms.

eq?

e.g. (eq? 'a 'a) => #t

(eq? 'c 3) => #f

(eq? '(a b) '(a b)) => #f

(define member

(lambda (x L)

(if (null? L)

#f

(if (eq? x (car L))

#t

(member? x (cdr L))))))

$$\begin{aligned} & (\text{member? } 'b \ '(a \ b \ c)) \\ &= (\text{member? } 'b \ '(b \ c)) \\ &= \#t \end{aligned}$$

Note that any further occurrences of  $b$  will not be found.

This function is tail-recursive because the last call to any function is to the function itself - recursion. Recursion means overhead for each function call. Tail-recursive functions can be optimized by replacing the recursive call with a loop. This is more efficient - we avoid the overhead of multiple calls.



We can rewrite (notA) recursive functions with a tail-recursive version and thus get efficient operation.

We will use a facility of Scheme to declare local functions.

[ We can write Scheme functions in 2 ways :

(define f (lambda (x y) ...))

alternatively, (define (f x y) ...)

Let's rewrite length in a tail-recursive version.

(define (length L)

(define (length1 L n)

(if (null? L)

n

(length1 (cdr L) (+ n 1))))

(length1 L 0))

e.g. (length '(a b c))

= (length1 '(a b c) 0)

= (length1 '(b c) (+ 0 1))

= (length1 '(c) 2)

= (length1 () 3)

= 3

The internal loop looks like this:

```
while true do
```

```
  (if (null? L) (return n))
```

```
  L := (cdr L)
```

```
  n := (+ n 1)
```

```
end;
```

There is an alternative way to write complex conditionals without nested if-then-else forms.

It's called cond (conditional)

```
(cond (-test1- -expr1-)
      (-test2- -expr2-)
      (-test3- -expr3-)
      (else -exprn-))
```

} clauses

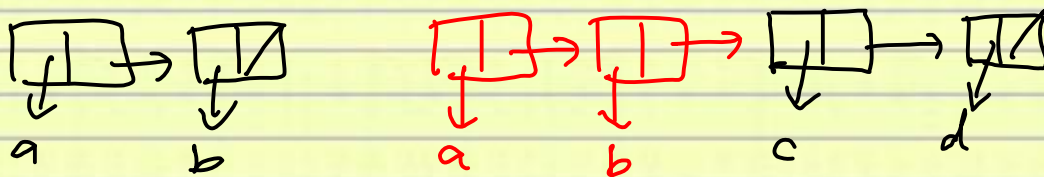
Let's rewrite member? using cond:

```
(define (member? x L)
  (cond ((null? L) #f)
        ((eq? x (car L)) #t)
        (else (member? x (cdr L)))))
```

3. Appending of two lists

e.g. (append '(a b) '(c d)) => (a b c d)

```
(define (append L1 L2)
  (if (null? L1)
      L2
      (cons (car L1) (append (cdr L1) L2)))))
```

$$\begin{aligned}
 & (\text{append } '(a\ b) \ '(c\ d)) \\
 &= (\text{cons } 'a \ (\text{append } '(b) \ '(c\ d))) \\
 &= (\text{cons } 'a \ (\text{cons } 'b \ (\text{append } () \ '(c\ d)))) \\
 &= (\text{cons } 'a \ (\text{cons } 'b \ '(c\ d))) \\
 &= (a\ b\ c\ d)
 \end{aligned}$$


So cons is a copy operation - we copy the first list and append it to the second list.