The Egg Cracking Problem

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Talk Overview

- Introduction
- The Egg Cracking Problem
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Introduction

- **What's AI?**
- **Commonsense reasoning**
- **Question answering**
- **The Common Sense problem page:**

  http://www-formal.stanford.edu/leora/cs

Commonsense Reasoning

- **Reasoning we do in our everyday lives.**

  Grapes are fruit.
  Bananas are fruit.
  Onions are neither grapes nor bananas.

- **What if we later learned the following information?**

  Onions are herbs.

- **Reasoning with incomplete information and the ability to retract previously drawn conclusions (non-monotonic reasoning).**

Question Answering

- $\Phi \vdash \alpha$

  Φ is a set of first-order formulae, or knowledge base, and α is a first-order formula, the question.

- **Theorem resolution**

Formalization Evaluation Criteria [1]

- **Epistemological adequacy**
- **Faithfulness to the real world**
- **Reusability**
- **Elaboration tolerance**

The Egg Cracking Problem

- **Description**

  A cook is cracking a raw egg against a glass bowl. Properly performed, the impact of the egg against the edge of the bowl will crack the eggshell in half. Holding the egg over the bowl, the cook will then separate the two halves of the shell with his fingers, enlarging the crack, and the contents of the egg will fall gently into the bowl. The end result is that the entire contents of the egg will be in the bowl, with the yolk unbroken, and that the two halves of the shell are held in the cook's fingers.

- **Questions**

  What happens if: The cook brings the egg to impact very quickly? Very slowly? The cook lays the egg in the bowl and exerts steady pressure with his hand? The cook, having cracked the egg, attempts to peel it off its contents like a hard-boiled egg? The bowl is made of loose leaf paper? of soft clay? The bowl is smaller than the egg? The bowl is upside down? The cook tries this procedure with a hard-boiled egg? With a coconut? With an M & M?

- **Credits**

  Contributed by Ernest Davis (davise@cs.nyu.edu), New York University, U.S.A. (18th September 1997)

A Formalization

- **There is a physically feasible single-agent plan for the egg cracking problem [1].**

- **Single agent plans can be defined inductively as follows:**

  1. if *act* is an action, then *act* is a plan;
  2. if *plan1* and *plan2* are plans, then $seq(plan1, plan2)$ is a plan;
  3. if *c* is a sentence of our language, and *plan1* and *plan2* are plans, then $cond(c, plan1, plan2)$ is a plan.

- **The physical feasibility of plan sequences is given by the following axiom:**

$$(physfeas(plan1, start(i)) \wedge (occurs(plan1, i) \Rightarrow physfeas(plan2, end(i)))) \Rightarrow physfeas(seq(plan1, plan2), start(i)) \quad (1)$$

- **For condition sequences, if c is true, then $plan1$ is physically feasible. Otherwise, $plan2$ is physically feasible.**

A Formalization (Cont.)

- **Assumptions**

$bowl(b)$ *(2)*

$upright(b, s)$ *(3)*

$empty(b, s)$ *(4)*

$hard(b)$ *(5)*

$WholeEgg(x, s)$ *(6)*

$raw(x, s)$ *(7)*

$above(loc, b, s)$ *(8)*

$capacity(b) \geq vol(x)$ *(9)*

- **Goal (feasibility)**

First we hit the egg against the bowl causing a crack. Then we quickly move the egg over the bowl. Next if the crack is facing up, we turn the crack so that it's facing down. Lastly, we open the shell of the egg.

$Physfeas(seq(a1, a2, a3, a4), s)$ *(10)*

where

$a1 = hit-against(eggshell(x), b, cf(eggshell(x), b))$ *(11)*

$a2 = qmove(x, loc)$ *(12)*

$a3 = cond(\neg down(gap(x, now)), qflip(x))$ *(13)*

$a4 = openshell(x, down)$ *(14)*

A Formalization (Cont.)

- **Goal (occurs)**

  First we hit the egg against the bowl causing a crack. Then we quickly move the egg over the bowl. Next if the crack is facing up, we turn the crack so that it's facing down. Lastly, we open the shell of the egg. If we can do all of this, then contents of the egg are still enclosed in the egg, which has a gap in it's shell, and the yolk is not broken.

$$occurs(seq(a1, a2, a3, a4), i) \Rightarrow (c1 \wedge c2 \wedge c3) \quad (15)$$

where

$$a1 = hit - against(eggshellof(x), b, cf(eggshellof(x), b)) \quad (16)$$
$$a2 = qmove(x, loc) \quad (17)$$
$$a3 = cond(\neg down(gap(x, now)), qflip(x)) \quad (18)$$
$$a4 = openshell(x, down) \quad (19)$$

and

$$c1 = enc - gs(yolkof(x), x, end(i)) \quad (20)$$
$$c2 = enc{-}gs(eggwhiteof(x), x, end(i)) \quad (21)$$
$$c3 = \neg broken(yolkof(x), end(i)) \quad (22)$$

A Formalization (Cont.)

- **Goal (opening the egg)**

  If the egg is cracked, it's contents are still intact, the egg is above the bowl, the bowl is large enough to hold the contents of the egg, and we open the egg, then the contents of the egg will fall into the bowl.

$$(c1 \wedge c2 \wedge c3 \wedge c4 \wedge c5 \wedge c6) \Rightarrow (r1 \wedge r2) \quad (23)$$

where

$$c1 = CrackedEgg(x, start(i)) \quad (24)$$
$$c2 = occurs(openshell(eggshellof(x), down), i) \quad (25)$$
$$c3 = enc-gs(yolkof(x), start(i)) \quad (26)$$
$$c4 = enc\text{–}gs(eggwhiteof(x), start(i)) \quad (27)$$
$$c5 = above(x, o, i) \wedge bowl(o) \wedge upright(o) \wedge empty(o, start(i)) \quad (28)$$
$$c6 = capacity(o) \geq vol(x) \quad (29)$$

and

$$r1 = enc-gs(yolkof(x), o, end(i)) \quad (30)$$
$$r2 = enc-gs(eggwhiteof(x), o, end(i)) \quad (31)$$

A Formalization (Cont.)

- **Salient properties of an egg:**

  A: *An egg has four states.*

  $$egg(x) \Leftrightarrow NotYetLaid(x, s) \vee WholeEgg(x, s) \vee CrackEgg(x, s) \vee BrokenEgg(x, s) \quad (32)$$

  A: *if the egg is whole or cracked, then the egg holds together*

  $$WholeEgg(x) \vee CrackedEgg(x, s) \Rightarrow together(x) \quad (33)$$

  A: *An egg is a package consisting of a egg yolk and egg white as long as the egg is whole or cracked.*

  $$(WholeEgg(x, i) \vee CrackedEgg(x, i)) \Rightarrow package(egginsideof(x), c, i) \quad (34)$$

  where

  $$c = \{eggyolk(x), eggwhite(x)\}.$$

A Formalization (Cont.)

- **The geometry an egg:**

  A: *The eggshell of a whole egg is a shell.*

  $$WholeEgg(x, i) \wedge eggshell(y, x) \Rightarrow shell(shape(y, i)) \quad (35)$$

  A: *The inside of the egg is the inside of a shell.*

  $$WholeEgg(x, i) \wedge egginsideof(y, x) \Rightarrow ShellInside(shape(y, i), shape(x, i)) \quad (36)$$

  T (35, 36, & Def.): *A whole egg is a filled shell.*

  $$WholeEgg(x, i) \Rightarrow FilledShell(shape(x, i)) \quad (37)$$

  A: *An object inside of another object is said to be enclosed by that object and vice-versa.*

  $$inside(shape(y, i), shape(x, i)) \Leftrightarrow enclosed(y, x, i) \quad (38)$$

  T (37 & Def.): *If an object is inside of a shell, then that object is enclosed by the shell.*

  $$shell(shape(x, i)) \wedge ShellInside(shape(y, i), shape(x, i)) \Rightarrow enclosed(y, x, i) \quad (39)$$

  T (35, 36, & 39): *The egg shell of a whole egg enclose the inside of an egg.*

  $$WholeEgg(x, i) \wedge eggshell(z, x) \wedge egginside(y, x) \Rightarrow enclosed(y, z, i) \quad (40)$$

A Formalization (Cont.)

- **Material of an egg:**

A: *Eggshells are breakable.*

$$eggshell(y, x) \Rightarrow breakable(y) \quad (41)$$

A: *If an object is breakable and you hit it against something hard, then the object will crack.*

$$breakable(x) \wedge hard(o) \wedge occurs(hitagainst(x, o, cf(x, o)), x) \Rightarrow cracked(x, end(i)) \quad (42)$$

A: *If the egg is cracked, then the egg is a cracked-egg.*

$$(egg(x) \wedge eggshell(x) \wedge cracked(y, i)) \Rightarrow CrackedEgg(x, i) \quad (43)$$

T (41, 42, & 43): *If you hit the egg against something that's hard with enough force, it will crack.*

$$(a1 \wedge a2 \wedge a3) \Rightarrow CrackedEgg(x, end(i)) \quad (44)$$

Where

$$a1 = WholeEgg(x, start(i)) \quad (45)$$

$$a2 = hard(o) \quad (46)$$

$$a3 = occurs(hitagainst(eggshellof(x), o, cf(eggshellof(x), i)), i)$$

A: *The raw egg white of is a liquid of high viscosity.*

$$(egg(x) \wedge eggwhite(w, x) \wedge raw(w, s)) \Rightarrow (liquid(w, s) \wedge (viscosity(w, s) = high)) \quad (47)$$

A: *Semisolid is solid.*

$$semisold(x, s) \Rightarrow solid(x, s) \quad (48)$$

A Formalization (Cont.)

- **Material of an egg (cont.):**

  A: *A yolk is not very narrow.*

  $$(egg(x) \wedge yolk(y, x) \wedge narrow(z)) \Rightarrow larger(y, z) \quad (49)$$

  A: *The yolk of a whole egg is never broken.*

  $$WholeEgg(x, s) \Rightarrow \neg BrokenYolk(yolkof(x), s) \quad (50)$$

  A: *The yolk and white of a raw egg are raw.*

  $$(egg(x) \wedge raw(x, s) \wedge yolk(y, x) \wedge eggwhite(w, x)) \Rightarrow (raw(y, s) \wedge raw(w, s)) \quad (51)$$

A Formalization (Cont.)

- **The initial crack:**

  A: *If a shell has a gap and a shell inside of it, then the gapped shell enclose another shell.*

  $$Shellgap(r1, r2) \wedge (shape(x, i) = r1) \wedge shell - inside(shape(y, i), r1) \Rightarrow enc - gs(y, x, i) \quad (52)$$

  A: *If a gapped shell encloses a package, then it does so to the contents of the package and vice-versa.*

  $$package(y, S, i) \Rightarrow enc - gs(y, x, i) \Leftrightarrow \forall s \in S \; enc - gs(s, x, i) \quad (53)$$

  A: *If a gapped shell is facing up and encloses another object, then it encloses that object as long as it's facing up.*

  $$(GappedShell(shape(x, i)) \wedge up(gap(x, i), i) \wedge enc - gs(y, x, start(i))) \Rightarrow enc - gs(y, x, end(i)) \quad (54)$$

A Formalization (Cont.)

- **The initial crack (cont.):**

  A: *If the object is solid and smaller than the gap, it will fall through the gap;*

  If the object is solid and larger than the gap, it will stay enclosed by the gapped shell;

  if the object is a liquid with high viscosity, the gap is narrow, and the time duration of the gap facing down is short, then the object will stay inside the gapped shell;

  If the viscosity is low in the previous then the object will fall through the gap.

  $a1 \Rightarrow c1 \wedge c2 \wedge c3 \wedge c4$ *(55)*

  where

  $a1 = GappedShell(shape(x), i) \wedge down(gap(x), i) \wedge enc-gs(o, x, start(i))$ *(56)*

  and

  $c1 = (solid(o) \wedge larger(o, gap(x, i))) \Rightarrow enc-gs(o, x, end(i))$ *(57)*

  $c2 = (solid(o) \wedge smaller(o, gap(x, i)) \wedge (duration(i) = long)) \Rightarrow fall-through(o, gap(x, i), i)$ *(58)*

  $c3 = (liquid(o) \wedge (viscosity(o) = high) \wedge (duration(i) = short) \wedge narrow(gap(x, i)) \Rightarrow enc-gs(o, x, end(i))$ *(59)*

  $(liquid(o) \wedge (viscosity(o) = low) \wedge (duration(i) = long)) \Rightarrow fall-through(o, gap(x, i), i)$ *(60)*

A Formalization (Cont.)

- **The initial crack (cont.):**

  T (53 & 55): *Likewise for the contents of the egg.*

  $$a1 \Rightarrow (\forall o \in S).(c1 \wedge c2 \wedge c3 \wedge c4) \quad (61)$$

  where

  $$a1 = GappedShell(shape(x), i) \wedge down(gap(x), i) \wedge enc-gs(y, x, start(i)) \wedge package(y, S, start(i)) \quad (62)$$

  and

  $$c1 = (solid(o) \wedge larger(o, gap(x, i))) \Rightarrow enc-gs(o, x, end(i)) \quad (63)$$

  $$c2 = (solid(o) \wedge smaller(o, gap(x, i)) \wedge (duration(i) = long)) \Rightarrow fall-through(o, gap(x, i), i) \quad (64)$$

  $$c3 = (liquid(o) \wedge (viscosity(o) = high) \wedge (duration(i) = short) \wedge narrow(gap(x, i)) \Rightarrow enc-gs(o, x, end(i)) \quad (65)$$

  $$(liquid(o) \wedge (viscosity(o) = low) \wedge (duration(i) = long)) \Rightarrow fall-through(o, gap(x, i), i) \quad (66)$$

A Formalization (Cont.)

- **The initial crack (cont.):**

A: *A cracked egg is a narrow gapped shell.*

$$CrackedEgg(x, i) \Rightarrow (GappedShell(shape(eggshellof(x), i)) \wedge narrow(gap(eggshellof(x), i))) \quad (67)$$

A: *During it's life a gapped shell will face only up or down throughout that time.*

$$Gappedshell(shape(x, i)) \Rightarrow \exists(i_1, \ldots, i_n).(c1 \wedge c2) \quad (68)$$

where

$$c1 = ([i_1, \ldots, i_n] = i) \quad (69)$$

$$c2 = (\forall i' \in \{i_1, \ldots, i_n\}(up(gap(x, i'), i')) \vee down(gap(x, i'), i')) \quad (70)$$

T (54, 67, & 68): *If moved quickly a cracked egg can be moved without the contents leaking out.*

$$(a1 \wedge a2 \wedge a3 \wedge a4 \wedge a5) \Rightarrow enc-gs(x, egginsideof(x), end(i)) \quad (71)$$

where

$$a1 = CrackedEgg(x, start(i)) \quad (72)$$

$$a2 = raw(x, i) \quad (73)$$

$$a3 = enc-gs(x, egginsideof(x), start(i)) \quad (74)$$

$$a4 = occurs(move(x, loc), i) \quad (75)$$

$$a5 = (duration(i) = short) \quad (76)$$

A Formalization (Cont.)

- **Opening up the egg:**

A: *If a gapped shell is open and it's above something, then it will fall onto that something.*

$$(occurs(openshell(x, down), i) \wedge above(x, o, i) \wedge enc-gs(y, x, start(i))) \Rightarrow occurs(fall-onto(y, o, i))) \quad (77)$$

A: *If an object falls onto a flat surface during some interval, it is on that surface at the end of that interval. On the other hand, if the object falls into a gapped shell (bowl) whose gap is up and which is not overly full, then the object will be enclosed by the gapped shell.*

$$occurs(fall-onto(x, o, i)) \Rightarrow (c1 \wedge c2) \quad (78)$$

where

$$c1 = (flat-surface(shape(o, i)) \Rightarrow on(x, i, end(i))) \quad (79)$$

$$c2 = (a1 \wedge a2 \wedge a3 \wedge a4) \Rightarrow enc-gs(y, o, end(i)) \quad (80)$$

$$a1 = Gappedshell(shape(o, i), i)$$

$$a2 = up(gap(o, i), i)$$

$$a3 = smaller(y, gap(o, i), i)$$

$$a4 = (capacity(o, start(i)) \geq (contents(o, start(i)) + vol(x)))$$

A: *A bowl is a gapped shell.*

$$bowl(o) \Rightarrow GappedShell(shape(o, s)) \quad (81)$$

A: *A an upright bowl has it's gap facing up.*

$$bowl(o) \wedge upright(o, s) \Rightarrow up(gap(o, s), s) \quad (82)$$

A Formalization (Cont.)

- **Opening up the egg (cont.):**

A: *Empty bowls have nothing in them.*

$$bowl(o) \wedge empty(o, s) \Rightarrow contents(o, s) = \varnothing \quad (83)$$

A: *If a package falls onto something, then the contents of that package fall onto that thing.*

$$(package(y, S, i) \wedge occurs(fall-onto(y, o), i)) \Rightarrow \forall x \in S\,(occurs(fall-onto(y, o), i)) \quad (84)$$

T (34, 55, 78, 81, 83, & 84): *If one opens up a cracked egg shell while the egg is over an upright bowl and the crack is turned down, and the bowl is empty, then the egg yolk and egg white are enclosed by the bowl.*

$$a1 \wedge a2 \wedge a3 \wedge a4 \wedge a5 \wedge a6 \Rightarrow c1 \wedge c2 \quad (85)$$

where

$$a1 = CrackedEgg(x, start(i)) \quad (86)$$
$$a2 = occurs(openshell(eggshellof(x), down), i) \quad (87)$$
$$a3 = enc-gs(yolkof(x), start(i)) \quad (88)$$
$$a4 = enc-gs(eggwhiteof(x), start(i)) \quad (89)$$
$$a5 = above(x, o, i) \wedge bowl(o) \wedge upright(o) \wedge empty(o, start(i)) \quad (90)$$

and

$$c1 = enc-gs(yolkof(x), o, end(i)) \quad (91)$$
$$c2 = enc-gs(eggwhiteof(x), o, end(i)) \quad (92)$$

A Formalization (Cont.)

- **Opening up the egg (proof 85: 34, 55, 78, 81, 83, & 84):**

  A34: *An egg is a package consisting of a egg yolk and egg white as long as the egg is whole or cracked.*

  A55: *If the object is solid and smaller than the gap, it will fall through the gap.*

  If the object is solid and larger than the gap, it will stay enclosed by the gapped shell.

  if the object is a liquid with high viscosity, the gap is narrow, and the time duration of the gap facing down is short, then the object will stay inside the gapped shell.

  If the viscosity is low in the previous, then the object will fall through the gap.

  A78: *If an object falls onto a flat surface during some interval, it is on that surface at the end of that interval. On the other hand, if the object falls into a gapped shell (bowl) whose gap is up (an upright bowl) and which is not overly full, then the object will be enclosed by the gapped shell.*

  A81: *A bowl is a gapped shell.*

  A83: *Empty bowls have nothing in them.*

  A84: *If a package falls onto something, then the contents of that package fall onto that thing.*

A Formalization (Cont.)

- **Feasibility of individual actions:**

  A: *A null action is feasible.*

  $Physfeas(null, s)$ *(93)*

  A: *To hit an object with a given force is feasible.*

  $Physfeas(hit-against(x, o, f), s)$ *(94)*

  A: *It is feasible that one can move an object if it is located in one place.*

  $together(x, s) \Rightarrow Physfeas(move(x, loc), s)$

  Def: *A quick move is a move which has a short duration.*

  $Occurs(qmove(x, loc), i) \Leftrightarrow (Occurs(move(x, loc), i) \wedge (duration(i) = short))$ *(95)*

  T: *It is feasible to move a cracked egg.*

  $CrackedEgg(x, s) \Rightarrow physfeas(qmove(x, loc), s)$ *(96)*

  A: *If an object is together, then it is feasible to flip it.*

  $together(x, s) \Leftrightarrow Physfeas(flip(x), s)$ *(97)*

A Formalization (Cont.)

- **Feasibility of individual actions (cont.):**

  Def.: *Flipping an object quickly is flipping an object within a short duration.*

  $$Occurs(qflip(x), i) \Leftrightarrow Occurs(flip(x), i) \wedge (duration(i) = short) \quad (98)$$

  T: *It is feasible to flip a cracked egg.*

  $$CrackedEgg(x, s) \Rightarrow Physfeas(qflip(x), s) \quad (99)$$

  A: *It is feasible to open up the shell of a raw cracked egg.*

  $$(a1 \wedge a2 \wedge a3) \Rightarrow Physfeas(openshell(x, down), s) \quad (100)$$

  Where

  $$a1 = CrackedEgg(x, s) \quad (101)$$
  $$a2 = raw(x, s) \quad (102)$$
  $$a3 = down(gap(eggshellof(x, s)), s) \quad (103)$$

A Formalization (Cont.)

- **Miscellaneous axioms on actions:**

A: *If an object is moved during an interval, then it's at the desired location at the end of the interval.*

$$Occurs(move(x, loc, i)) \Rightarrow at(x, loc, end(i)) \quad (104)$$

A: *If two objects are at a location and one of them is above the location, then it is also above the other object.*

$$(at(x, loc, s) \wedge above(loc, o, s)) \Rightarrow above(x, o, s) \quad (105)$$

A: *If a gapped shell is facing up, then flipping it makes it face downward.*

$$(a1 \wedge a2 \wedge a3) \Rightarrow down(gap(o, i), end(i))$$

where

$$a1 = Gappedshell(shape(o, start(i))) \quad (106)$$

$$a2 = up(gap(o, i), start(i)) \quad (107)$$

$$a3 = occurs(flip(o), i) \quad (108)$$

A Formalization (Cont.)

- **Frame axioms describe how the world stays the same [3].**

- **Example**

$$\forall(a, x, s).((Holding(x, s) \wedge \neg(a = Release)) \Rightarrow Holding(x, Result(a, s))) \quad (109)$$

Conclusion

- **These formalizations seems to be an end result.**

- **One characterization of reasoning is that we either reason based upon stored knowledge (experiences, memories, etc.) or that we reason without the use of stored knowledge [4,5].**

- **Are there situations where we use both forms of reasoning? What about structural alignment?**

Conclusion (cont.)

- **Consider the following axiom schema:**

$$A(\Phi) \wedge \forall x'(\Phi(x') \Rightarrow P(x')) \Rightarrow \forall x'(P(x') \Rightarrow \Phi(x')) \qquad (110)$$

A is a finite set of first-order formulae (i.e., a theory).
$P(x')$ is a predicate appearing in A.
$A(\Phi)$ is the result of replacing all occurrences of $P(x'))$ in A with Φ.

- **Without examples it is difficult to apply.**

References

[1] Morgenstern, L., Foundation of a Logic of Knowledge, Action, and Communication, *Ph.D. thesis*, NYU, 1988.

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[5] Kurtz, K.J., Genter, D., and Gunn, V., Reasoning, *Cognitive Science*, 1999, Pages 145-200.