Dynamical Systems and Conley’s Topological Index

Ibrahim Jawarneh,
Advisor: Professor Ross Staffeldt
NMSU - Mathematical Sciences

March 17, 2017

A dynamical system is a flow $\varphi : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$ which is a function characterized by the properties $\varphi(\varphi(x, t), s) = \varphi(x, t + s)$ and $\varphi(x, 0) = x$. Intuitively, looking at one point $x$, $\varphi(x, t)$ provides a picture of the point $x$ moving as time $t$ changes. Usually a flow is obtained from a system of differential equations $\frac{dx}{dt} = f(x)$ by integration. Fixed points of a flow are points $x_0$ such that $\varphi(x_0, t) = x_0$ for all time. They may be found by solving the equation $f(x_0) = 0$. Classification theorems of isolated fixed points depend on eigenvalues of the linearization of the dynamical system at the fixed point, but not all isolated fixed points may be classified by this method. Moreover, this method is defined only near isolated fixed points.

Generalizing the idea of an isolated fixed point of a flow is the idea of an isolated invariant set. A subset $S \subset \mathbb{R}^n$ is invariant if for all $x \in S$, $\varphi(x, t) \in S$ for all $t$. Conley’s topological index of a compact isolated invariant set uses the language of algebraic topology to define a generalization of the classification an isolated fixed point. In this presentation, we present examples to introduce the theory and describe applications.