New Results on the Asymptotic Behavior of Solutions to Some Second Order Nonhomogeneous Difference Equations

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Abstract

We investigate the asymptotic behavior of solutions to the following system of second order nonhomogeneous difference equation:

\[
\begin{cases}
    u_{n+1} - (1 + \theta_n)u_n + \theta_n u_{n-1} & \in c_n Au_n + f_n & n \geq 1 \\
    u_0 = x, & \sup_{n \geq 0} |u_n| < +\infty
\end{cases}
\]

where \( A \) is a maximal monotone operator in a real Hilbert space \( H \), \( \{c_n\} \) and \( \{\theta_n\} \) are positive real sequences and \( \{f_n\} \) is a sequence in \( H \). We show the weak and strong convergence of solutions and their weighted averages to an element of \( A^{-1}(0) \), which is the asymptotic center of the sequence \( \{u_n\} \), under appropriate assumptions on the sequences \( \{c_n\} \), \( \{\theta_n\} \) and \( \{f_n\} \). Our results continue our previous work in [13,17], by presenting some new results on the asymptotic behavior of solutions, including in particular a completely new strong convergence result, and extend some previous results by Apreutesei [3,4], Morosanu [26] and Mitidieri-Morosanu [22] to the nonhomogeneous case and without assuming \( A \) to have a nonempty zero set.