Single View Computer Vision in Polyhedral World: Geometric Inference and Performance Characterization

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Abstract

An algorithm for making consistent 2-D to 3-D geometric inference in polyhedral world using one perspective line drawing is described. Hypotheses are made on the internal angles of visible faces. The normals to the face planes are then determined. Valid normals lead to the reconstruction of the 3-D polyhedral world up to a scale factor. The performance of the algorithm is verified by using covariance matrix propagation. The experimental results show satisfactory performance.

The general propagation formulae for the covariance matrix of both observed and inferred quantities are also derived.

1. Introduction

Given only one 2-D perspective projection line drawing of a 3-D polyhedral world and the camera calibration, without any other hints, except that the faces of polyhedra are planar, what can be inferred about the polyhedral world, in terms of the geometric properties of the polyhedra? How accurate is the inference? These are two aspects of a classical computer vision problem. The goal is to find a possible explanation of the polyhedral world which would be consistent with the observed 2-D image. Liebowitz *et al* [3] and Shum *et al* [4] proposed methods to solve the single view computer vision problem. Both authors took advantage of geometric regularity in the real world, such as parallelism and orthogonality. In our approach, the polyhedra can have general shapes, not limited to those with parallelism or orthogonality.

Hypotheses are made on the internal angles of visible faces. The normals to the face planes are then determined. Valid normals lead to the reconstruction of the 3-D polyhedral world up to a scale factor.

The validity of the hypotheses is tested using the covariance matrix associated with the solution, which is derived analytically starting from the covariance matrix of the observed quantities and is propagated through each inference step. Performance validation is important especially when non-deterministic algorithms are involved because the behavior of such algorithms can be at most probabilistically predicted, but not logically.

2. Covariance Matrix of Both Observed and Inferred Quantities

Let $\Sigma_{\Delta\vec{X}} \in \mathbb{R}^{n\times n}$ be the covariance matrix of $\vec{X}$, a vector of observed perspective projection locations and line directions, and $\Sigma_{\Delta\vec{\Theta}} \in \mathbb{R}^{m\times m}$ be the covariance matrix of $\vec{\Theta}$, a vector of inferred 3-D polyhedral geometry. The covariance propagates from $\Sigma_{\Delta\vec{X}}$ to $\Sigma_{\Delta\vec{\Theta}}$. Haralick [1] summarized the methodology of covariance propagation for a vision algorithm from an observed data vector $\vec{X} \in \mathbb{R}^n$ to an inferred parameter vector $\vec{\Theta} \in \mathbb{R}^m$. As assumed in [1], the solution to the vision algorithm can be modeled as optimization of certain objective functions having finite second partial derivatives and the perturbations are small enough so that a first order approximation is good enough.

Here, we are interested in the covariance matrix $\Sigma_{\vec{\Theta},\vec{X}} \in \mathbb{R}^{(n+m)\times(n+m)}$, which gives the covariance between $\vec{\Theta}$ and $\vec{X}$ as well as $\Sigma_{\Delta\vec{\Theta}}$ and $\Sigma_{\Delta\vec{X}}$. One such situation happens when inference is not being made in the first step but has intermediate steps and $\vec{X}$ is an inferred quantity from a previous step rather than a directly observed quantity. $\vec{\Theta}$ is further inferred from $\vec{X}$. Since they are all inferred quantities, it is likely that the covariance between $\vec{\Theta}$ and $\vec{X}$ would be of interest. In the following sections, we will derive the closed-form formula to calculate the covariance matrix $\Sigma_{\vec{\Theta},\vec{X}}$.

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2.1. Explicit Function $\vec{\Theta} = \mathbf{F}(\vec{X})$ Is Known

Given the explicit function $\vec{\Theta} = \mathbf{F}(\vec{X}) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, $\vec{\Theta}$ is calculated by evaluating $\mathbf{F}(\vec{X})$, which is a vector function in general. Taking the first order Taylor series expansion, we get $\Delta\vec{\Theta} \approx \frac{d\mathbf{F}(\vec{X})^T}{d\vec{X}}\Delta\vec{X}$. So the covariance matrix can be found as

$$\Sigma_{\Delta\vec{\Theta},\Delta\vec{X}} \approx \begin{bmatrix} \mathbf{G}^T\Sigma_{\Delta\vec{X}}\mathbf{G} & \mathbf{G}^T\Sigma_{\Delta\vec{X}} \\ \Sigma_{\Delta\vec{X}}\mathbf{G} & \Sigma_{\Delta\vec{X}} \end{bmatrix} \quad (1)$$

where $\mathbf{G} = \frac{d\mathbf{F}(\vec{X})}{d\vec{X}}$. The upper-left sub-matrix is $\Sigma_{\Delta\vec{\Theta}}$.

2.2. Unconstrained Optimization with Objective Function $F(\vec{X},\vec{\Theta})$

$F(\vec{X},\vec{\Theta}) : (\mathbb{R}^n,\mathbb{R}^m) \rightarrow \mathbb{R}^+ \cup \{0\}$ is a nonnegative scalar function of $\vec{X}$ and $\vec{\Theta}$. $\vec{\Theta}$ is the optimal solution to the minimization problem: $\min_{\vec{\Theta}\in\mathbb{R}^m} F(\vec{X},\vec{\Theta})$. From [1], $\Delta\vec{\Theta} \approx -\left(\frac{\partial\vec{g}}{\partial\vec{\Theta}}\right)^{-1}\frac{\partial\vec{g}^T}{\partial\vec{X}}\Delta\vec{X}$, where $\vec{g} = \frac{\partial F(\vec{X},\vec{\Theta})}{\partial\vec{\Theta}}$. It follows that $\Sigma_{\Delta\vec{\Theta},\Delta\vec{X}}$ is

$$\begin{bmatrix} H^{-1}M^T\Sigma_{\Delta\vec{X}}MH^{-1} & -H^{-1}M^T\Sigma_{\Delta\vec{X}} \\ -\Sigma_{\Delta\vec{X}}MH^{-1} & \Sigma_{\Delta\vec{X}} \end{bmatrix} \quad (2)$$

where $H = \frac{\partial\vec{g}}{\partial\vec{\Theta}}$ and $M = \frac{\partial\vec{g}}{\partial\vec{X}}$. The upper-left sub-matrix is $\Sigma_{\Delta\vec{\Theta}}$.

2.3. Constrained Optimization with Objective Function $F(\vec{X},\vec{\Theta})$ and constraints $\mathbf{s}(\vec{\Theta})$

$F(\vec{X},\vec{\Theta}) : (\mathbb{R}^n,\mathbb{R}^m) \rightarrow \mathbb{R}^+ \cup \{0\}$ is a nonnegative scalar function of $\vec{X}$ and $\vec{\Theta}$. $\mathbf{s}(\vec{\Theta}) = 0$ represents a set of equations constraining $\vec{\Theta}$. These constraints can be equalities or inequalities. $\vec{\Theta}$ is the optimal solution to the minimization problem $\min_{\mathbf{s}(\vec{\Theta})} F(\vec{X},\vec{\Theta})$. From [1],

$$\begin{bmatrix} \frac{\partial\vec{g}}{\partial\vec{\Theta}} & \frac{\partial\mathbf{s}}{\partial\vec{\Theta}} \\ \frac{\partial\mathbf{s}^T}{\partial\vec{\Theta}} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \Delta\vec{\Theta} \\ \Delta\vec{\Lambda} \end{bmatrix} \approx \begin{bmatrix} -\frac{\partial\vec{g}^T}{\partial\vec{X}} \\ \mathbf{0} \end{bmatrix}\Delta\vec{X} \quad (3)$$

where $\vec{g} = \frac{\partial F(\vec{X},\vec{\Theta})}{\partial\vec{\Theta}}$ and $\vec{\Lambda}$ is the Lagrangian multiplier vector. Transforming Eq (3) to

$$\begin{bmatrix} \frac{\partial\vec{g}}{\partial\vec{\Theta}} & \frac{\partial\mathbf{s}}{\partial\vec{\Theta}} & \mathbf{0} \\ \frac{\partial\mathbf{s}^T}{\partial\vec{\Theta}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}\begin{bmatrix} \Delta\vec{\Theta} \\ \Delta\vec{\Lambda} \\ \Delta\vec{X} \end{bmatrix} \approx \begin{bmatrix} -\frac{\partial\vec{g}^T}{\partial\vec{X}} \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix}\Delta\vec{X}$$

and letting

$$C = \begin{bmatrix} \frac{\partial\vec{g}}{\partial\vec{\Theta}} & \frac{\partial\mathbf{s}}{\partial\vec{\Theta}} & \mathbf{0} \\ \frac{\partial\mathbf{s}^T}{\partial\vec{\Theta}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad D = \begin{bmatrix} -\frac{\partial\vec{g}^T}{\partial\vec{X}} \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix}$$

we obtain the covariance matrix (C is symmetric)

$$\Sigma_{\Delta\vec{\Theta},\Delta\vec{\Lambda},\Delta\vec{X}} \approx C^{-1}D\Sigma_{\Delta\vec{X}}D^TC^{-1} \quad (4)$$

3. Geometric Inference and Covariance Propagation in the Polyhedral World

Given one 2-D perspective line-drawing of a polyhedral world, infer the 3-D positions and orientations of each polyhedron whose perspective projection can result in a perspective projected line-drawing statistically consistent with the observed 2-D perspective line-drawing. Validate the inference procedure by comparing the covariance matrix derived analytically and the one estimated experimentally.

Hypotheses are made about intrinsic properties of a polyhedron, such as the *length* of one edge of each polyhedron and/or the *angle* between two edges, while those geometric properties that change over space, such as the 3-D *position* of a vertex, are avoided.

A polyhedron has planar faces, which is a strong constraint in the problem. The inference procedure starts with hypotheses of angles and the length of one edge of each polyhedron, and repeats with new hypotheses until consistency is attained.

Polyhedra having from some view the same topology but incongruent geometric structure might produce the same perspective images, even if we cannot come up with the "correct" values of the unknowns, we might arrive at a solution which produces the same perspective image. This is perfectly good up to the consistency requirement, since a single 2-D perspective image is not enough to make 3-D inference in general.

We assume that the visible faces, edges and vertices have been labeled and grouped in the given perspective image. They are represented as: Π_k for a visible face, with normal $\vec{p}_k$ and a 3-D point $\mathbf{W}_k$ on the face, $k = 1,...,K$; L_i for a visible edge with direction cosine vector $\vec{b}_i$, two terminal points $\mathbf{P}_{i1}$ and $\mathbf{P}_{i2}$ and the observation plane normal $\vec{n}_i$, $i = 1,...,V$; $\mathbf{P}_q$ for a visible vertex with perspective projection $\mathbf{P}_q^* = (u_q, v_q, f)^T$, $q = 1,...,Q$, where u_q and v_q are the projected coordinates of $\mathbf{P}_q$ and f is the distance the imaging plane is in front of the center of projection. To simplify the notation, we use:

$$\underline{\mathbf{P}} = (\mathbf{P}_1^T,\cdots,\mathbf{P}_Q^T)^T \quad \underline{\mathbf{b}} = (\vec{b}_1^T,\cdots,\vec{b}_V^T)^T$$
$$\underline{\mathbf{p}} = (\vec{p}_1^T,\cdots,\vec{p}_K^T)^T \quad \underline{\mathbf{W}} = (\mathbf{W}_1^T,\cdots,\mathbf{W}_K^T)^T$$
$$\underline{\mathbf{n}} = (\vec{n}_1^T,\cdots,\vec{n}_V^T)^T \quad \underline{\mathbf{uv}} = (u_1,v_1,\cdots,u_Q,u_Q)^T$$

The initial covariance matrix of the observed quantities is $\Sigma_{\underline{\mathbf{uv}}}$ and the covariance matrix of the unknowns is $\Sigma_{\underline{\mathbf{P}},\underline{\mathbf{b}},\underline{\mathbf{p}},\underline{\mathbf{W}}}$.

Step 1. Calculating the normal to the observation plane

Goal: The normal $\vec{n}_i$ to the observation plane of each edge L_i. The observation plane of a line passes the line and the center of projection.
Observations: The 2-D perspective image $\mathbf{P}^*_{i1}\mathbf{P}^*_{i2}$ of each 3-D edge $\mathbf{P}_{i1}\mathbf{P}_{i2}, i = 1, ..., V$.
Inference: $\vec{n}_i = \frac{\mathbf{P}^*_{i1} \times \mathbf{P}^*_{i2}}{\|\mathbf{P}^*_{i1} \times \mathbf{P}^*_{i2}\|}$, $i = 1, .., V$
Covariance Propagation: In this step, we have explicit formula for $\underline{\mathbf{n}}$, so by Eq (1) we know

$$\Sigma_{\underline{\mathbf{n}},\underline{\mathbf{uv}}} \approx \begin{bmatrix} \mathbf{G}_1^T \Sigma_{\underline{\mathbf{uv}}} \mathbf{G}_1 & \mathbf{G}_1^T \Sigma_{\underline{\mathbf{uv}}} \\ \Sigma_{\underline{\mathbf{uv}}} \mathbf{G}_1 & \Sigma_{\underline{\mathbf{uv}}} \end{bmatrix}$$

where

$$\mathbf{G}_1 = \frac{\partial \left[\left(\frac{\mathbf{P}^*_{11} \times \mathbf{P}^*_{12}}{\|\mathbf{P}^*_{11} \times \mathbf{P}^*_{12}\|} \right)^T, \cdots, \left(\frac{\mathbf{P}^*_{V1} \times \mathbf{P}^*_{V2}}{\|\mathbf{P}^*_{V1} \times \mathbf{P}^*_{V2}\|} \right)^T \right]^T}{\partial \underline{\mathbf{uv}}}$$

Step 2. From coplanar edges with known in-between angles on a face to infer the normal to the face

Suppose there are N coplanar edges $L_1, L_2, ..., L_N$, with unknown direction cosine $\vec{b}_1, \vec{b}_2, ..., \vec{b}_N$ on the face Π_k. The normal $\vec{p}_k$ to Π_k and the position of Π_k are unknown.
Goal: Find the normal $\vec{p}_k$ to face Π_k, $k = 1, ..., K$.
Inferred Quantities from Previous Steps: $\vec{n}_1, \vec{n}_2, ..., \vec{n}_N$, with covariance matrix $\Sigma_{\vec{n}_1,...,\vec{n}_N}$.
Assumptions: Hypothesized M_k cosines of the angles between M_k ($M_k \leq N$) pairs of the N coplanar lines. We write these assumptions as

$$|\gamma_1| = |\vec{b}_1 \cdot \vec{b}_2|, \cdots, |\gamma_{M_k-1}| = |\vec{b}_{M_k-1} \cdot \vec{b}_{M_k}|, |\gamma_{M_k}| = |\vec{b}_{M_k} \cdot \vec{b}_1|$$

The angle cosines that are most likely to be observed in the real world should be hypothesized [4].
Inference: For the i-th edge L_i, we know: $\vec{b}_i \perp \vec{n}_i$ and $\vec{b}_i \perp \vec{p}_k$. So $\vec{b}_i = \frac{\vec{n}_i \times \vec{p}_k}{\|\vec{n}_i \times \vec{p}_k\|}$. Hence, for $i = 1, ..., M_k$,

$$|\gamma_i| = |\vec{b}_i \cdot \vec{b}_{i+1}| = \left| \frac{\vec{n}_i \times \vec{p}_k}{\|\vec{n}_i \times \vec{p}_k\|} \cdot \frac{\vec{n}_{i+1} \times \vec{p}_k}{\|\vec{n}_{i+1} \times \vec{p}_k\|} \right| \quad (5)$$

To solve for $\vec{p}_k$, we need two more equations, since we already have a constraint $\|\vec{p}_k\| = 1$. When there are more than two equations, the equations can be solved by constrained optimization. Let

$$f(\vec{n}_i, \vec{n}_{i+1}, \vec{p}_k, \gamma_i) = \begin{cases} [(\vec{n}_i \times \vec{p}_k) \cdot (\vec{n}_{i+1} \times \vec{p}_k)]^2 \\ -(\gamma_i \|\vec{n}_i \times \vec{p}_k\| \|\vec{n}_{i+1} \times \vec{p}_k\|)^2, & \gamma_i \neq 0 \\ (\vec{n}_i \times \vec{p}_k) \cdot (\vec{n}_{i+1} \times \vec{p}_k), & \gamma_i = 0 \end{cases}$$

and the constrained optimization can be formulated as a non-linear least squares problem:

$$\min_{\|\vec{p}_k\|=1} F(\vec{n}_1, ..., \vec{n}_{M_k}, \vec{p}_k) = \min_{\|\vec{p}_k\|=1} \sum_{i=1}^{M_k} f^2(\vec{n}_i, \vec{n}_{i+1}, \vec{p}_k, \gamma_i)$$

The above problem can be solved numerically. I random guesses are made for the initial value of each $\vec{p}_k$ and save the first J ($J \leq I$) most optimal solutions for later use in hypothesis test. I and J are determined by training experiments. After the numerical optimization, the consistency is checked for the direction cosines of edges which can be derived from different paths. The best combination of the solutions of the normals which makes the direction cosines of edges most consistent is selected. The consistency is calculated by the summation of the squares of difference of direction cosines of edges which can be computed by two different faces, that is, to find the best combination of $\vec{p}_k$, from the optimal and sub-optimal solutions which minimizes

$$\begin{aligned} \epsilon &= \sum_{i=1}^{V} \min\{\|\vec{b}_{i1} - \vec{b}_{i2}\|^2, \|\vec{b}_{i1} + \vec{b}_{i2}\|^2\} \\ &= \sum_{i=1}^{V} \min \left\{ \left\| \frac{\vec{n}_i \times \vec{p}_{i1}}{\|\vec{n}_i \times \vec{p}_{i1}\|} - \frac{\vec{n}_i \times \vec{p}_{i2}}{\|\vec{n}_i \times \vec{p}_{i2}\|} \right\|^2, \right. \\ &\quad \left. \left\| \frac{\vec{n}_i \times \vec{p}_{i1}}{\|\vec{n}_i \times \vec{p}_{i1}\|} + \frac{\vec{n}_i \times \vec{p}_{i2}}{\|\vec{n}_i \times \vec{p}_{i2}\|} \right\|^2 \right\} \end{aligned}$$

where $\vec{p}_{i1}$ and $\vec{p}_{i2}$ are the normals to the two visible faces which intersect at edge L_i.
Covariance Propagation: $\Sigma_{\underline{\mathbf{p}};\lambda_1,...,\lambda_K;\underline{\mathbf{n}}} \approx C^{-1} D \Sigma_{\underline{\mathbf{n}}} D^T C^{-1}$, where

$$C = \begin{bmatrix} \left(\frac{\partial \vec{g}}{\partial \underline{\mathbf{p}}}\right)_{3K \times 3K} & \left(\frac{\partial \vec{s}}{\partial \underline{\mathbf{p}}}\right)_{3K \times K} & \mathbf{0}_{3K \times 3V} \\ \left(\frac{\partial \vec{s}}{\partial \underline{\mathbf{p}}}\right)^T_{K \times 3K} & \mathbf{0}_{K \times K} & \mathbf{0}_{K \times 3V} \\ \mathbf{0}_{3V \times 3K} & \mathbf{0}_{3V \times K} & \mathbf{I}_{3V \times 3V} \end{bmatrix}$$

$$D = \begin{bmatrix} -\frac{\partial \vec{g}^T}{\partial \underline{\mathbf{n}}_{3K \times 3V}} \\ \mathbf{0}_{K \times 3V} \\ \mathbf{I}_{3V \times 3V} \end{bmatrix}$$

$$\vec{g} = \frac{\partial \sum_{k=1}^{K} F(\vec{n}_{k1}, ..., \vec{n}_{kM_k}, \vec{p}_k)}{\partial \underline{\mathbf{p}}},$$

$$\vec{s} = [\|\vec{p}_1\|^2 - 1, \cdots, \|\vec{p}_K\|^2 - 1]^T$$

Step 3. From an edge on a face with known normal to infer the direction cosine of the edge

Goal: $\vec{b}_i$, $i = 1, ...V$
Inferred Quantities from Previous Steps: $\vec{p}_k$, $k = 1, ..., K$, $\vec{n}_i$, $i = 1, ...V$
Inference: $\vec{b}_i = \frac{\vec{n}_i \times \vec{p}_k}{\|\vec{n}_i \times \vec{p}_k\|}$ $(i = 1)$

Step 4. From the length of an edge and its direction cosine to infer its 3-D position

Goal: The terminal points of the edge, $\mathbf{P}_{j1}, \mathbf{P}_{j2}$.

Observations: $\mathbf{P}^*_{j1} = (u_{j1}, v_{j1}, f)^T, \mathbf{P}^*_{j2} = (u_{j2}, v_{j2}, f)^T$
Inferred Quantities from Previous Steps: $\vec{b}_j, \quad j = 1.$
Assumptions: The length d of the j-th edge L_j of a polyhedron. The value of d does not affect the other inference results up to the scale.
Inference: See [2] p63 for details.

Step 5. From a known edge on a face to infer the 3-D position of the face

Goal: A point $\mathbf{W}_k$ on the face.
Inferred Quantities from Previous Steps: The k-th face normal $\vec{p}_k$. The terminal points of an edge $\mathbf{P}_{j1}, \mathbf{P}_{j2}$.
Inference: $\mathbf{W}_k = (\mathbf{P}_{j1} + \mathbf{P}_{j2})/2$, the geometric mid-point between $\mathbf{P}_{j1}$ and $\mathbf{P}_{j2}$, gives the best solution in the least square sense.

Step 6. From a known face to infer the 3-D positions of edges on this face

Goal: The terminal points of the i-th edge L_i, $\mathbf{P}_{i1}$ and $\mathbf{P}_{i2}$
Observations: $\mathbf{P}^*_{i1}$ and $\mathbf{P}^*_{i2}$
Inferred Quantities from Previous Steps: $\vec{p}_k$ and $\mathbf{W}_k$ of the face on which the edge is located.
Inference: $\mathbf{P}_{i1}$ and $\mathbf{P}_{i2}$ satisfy $(\mathbf{P} - \mathbf{W}_k)^T \vec{p}_k = 0$ and by perspective projection $\mathbf{P}_{i1} = \eta_{i1}\mathbf{P}^*_{i1}$ and $\mathbf{P}_{i2} = \eta_{i2}\mathbf{P}^*_{i2}$. Solving these equations, we obtain $\mathbf{P}_{i1} = \frac{\mathbf{W}_k^T \vec{p}_k}{\mathbf{P}^{*T}_{i1} \vec{p}_k}\mathbf{P}^*_{i1}$ and $\mathbf{P}_{i2} = \frac{\mathbf{W}_k^T \vec{p}_k}{\mathbf{P}^{*T}_{i2} \vec{p}_k}\mathbf{P}^*_{i2}$.

After the edge is known, go back to Step 5 to infer the position of the other face of the edge. Step 5 and step 6 form a recursive call loop. The recurrence keeps going until all the edges and faces of each polyhedron are resolved.

4. Experiment Results

We built a polyhedral world composed of a cuboid and a pyramid. Gaussian noise is applied to generate noisy observation. The noise is not required to be Gaussian as long as it has zero mean and finite variance. Fig 1 shows the perspective projection of the inferred 3-D world with noise standard deviation at 0.005 and 0.1.

All the non-deterministic behavior introduced by the algorithm comes from Step 2 – the numerical optimization whose result depends on initial guess. Hence, we concentrate on finding the covariance matrix about $\underline{\mathbf{p}}$ by Step 1 and 2. (The covariances of other geometric properties inferred from Step 3 to 6 are omitted since these steps are deterministic.) Corresponding entries in the analytical derived covariance matrix and the experiment-estimated one for $\underline{\mathbf{p}}$ show similar values and also form similar value patterns. (See *http://isl.ee.washington.edu/~msong/covariance.pdf*)

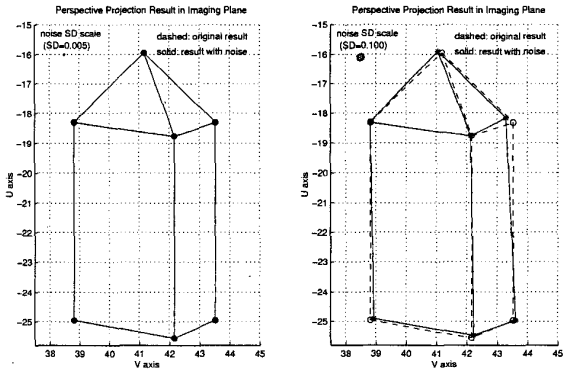

Figure 1. Perspective Projection of Inferred $3-D$ Polyhedral World

5. Conclusions

We gave a general approach to making geometric inference from 2-D to 3-D in a polyhedral world using a single view perspective projection. We verified our implementation by comparing the estimated covariance matrix and the analytically computed covariance matrix. They showed consistent characteristics when the noise is small, additive and finite in variance.

The procedure outlined in this paper provides a way to begin for the covariance matrices of the observed 2-D perspective projection points and propagate these covariances to corresponding 3-D polyhedral vertices. The inferred 3-D positions are not those that have the smallest covariance. In another paper, we will discuss this more difficult problem.

For non-deterministic algorithms, *e.g.* some optimization problems whose solution depends on the choice of the initial value, the logical proof of correctness of a program is impossible. Off-line Monte Carlo testing has been the only way to test the correctness of the program. However, the on-line hypothesis testing using covariance matrix provides another way to safeguard the solution.

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