Example of B+ tree

B+ Tree Example

- A balanced tree
- Each node can have at most *m* key fields and *m+1* pointer fields
- Half-full must be satisfied (except root node):
- m is even and m=2d
  - Leaf node half full: at least d entries
  - Non-leaf node half full: at least d entries
- m is odd and m = 2d+1
  - Leaf node half full: at least d+1 entries
  - Non-leaf node half full: at least d entries (i.e., d+1 pointers)

Show the tree after insertions

- Suppose each B+-tree node can hold up to 4 pointers and 3 keys.
- m=3 (odd), d=1
- Half-full (for odd m value)
  - Leaf node, at least 2 (d+1) entries
  - Non-leaf nodes, at least 2 (d+1) pointers (1 entry)
- Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

- Insert 1

1		

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

1		

- Insert 3, 5

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

1		

- Insert 3, 5

1	3	5

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

1	3	5

- Insert 7

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

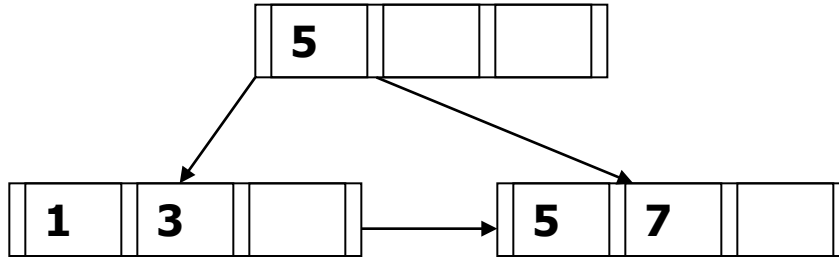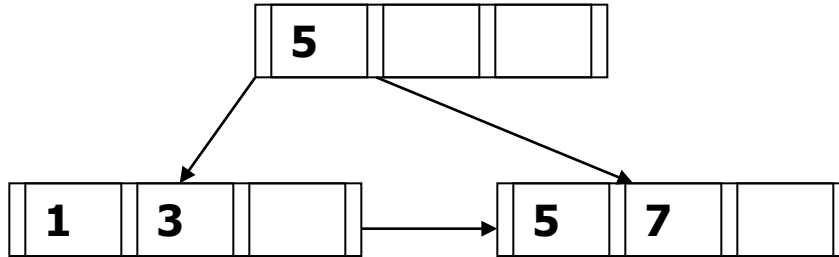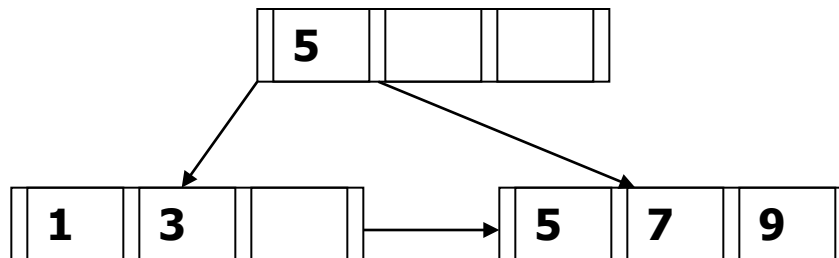| 1 | 3 | 5 | |

- Insert 7

| 5 | | |

| 1 | 3 | | → | 5 | 7 | |
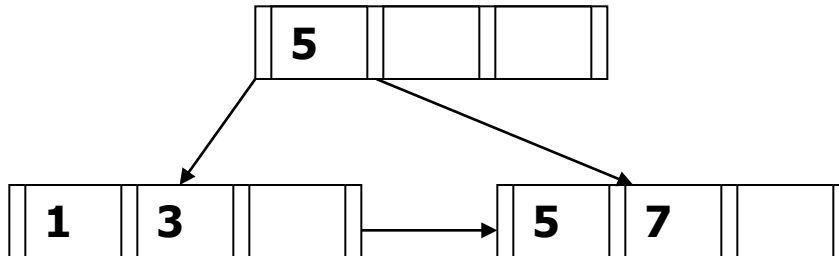
Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

- Insert 9

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

```
        [ 5 |   |   ]
        /         \
[ 1 | 3 |   ] → [ 5 | 7 |   ]
```

- Insert 9

```
        [ 5 |   |   ]
        /         \
[ 1 | 3 |   ] → [ 5 | 7 | 9 ]
```

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

- Insert 2
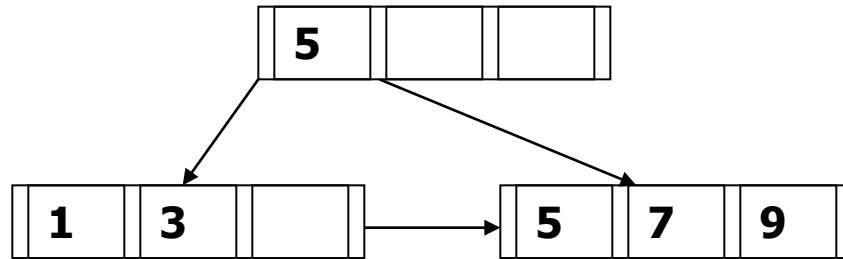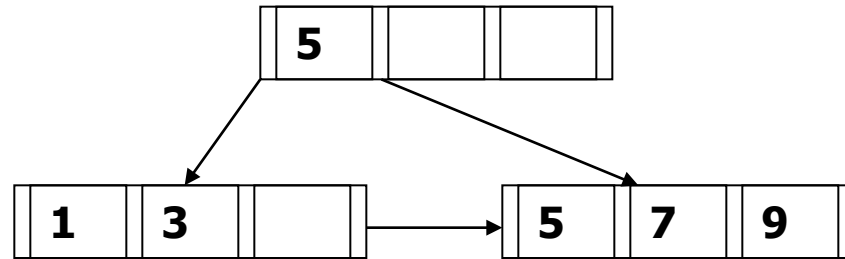
Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10
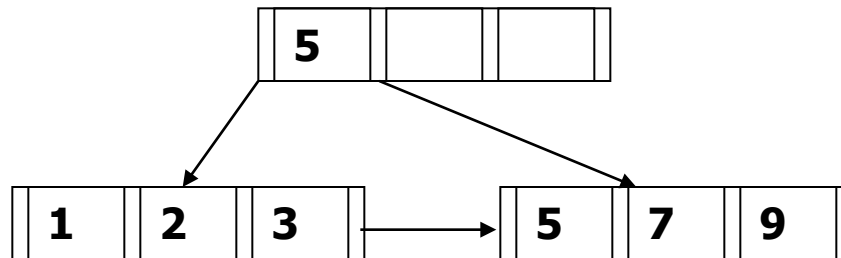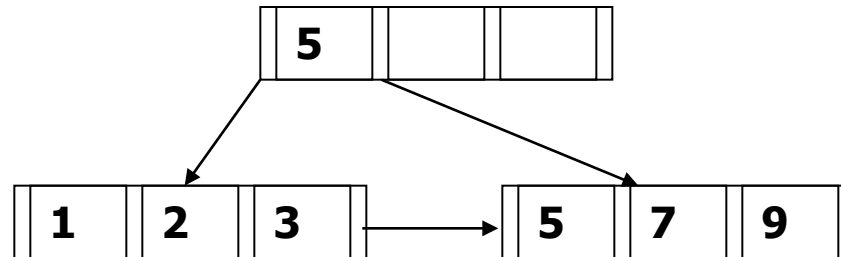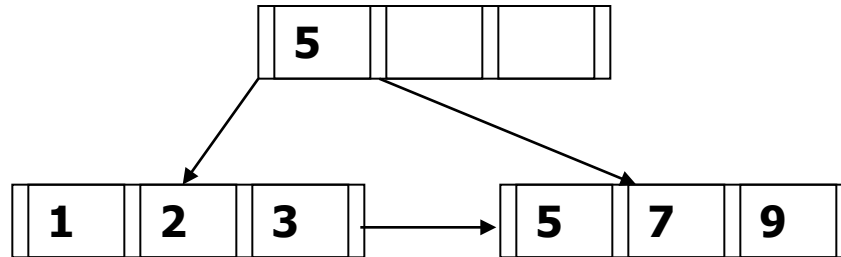

- Insert 2

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10
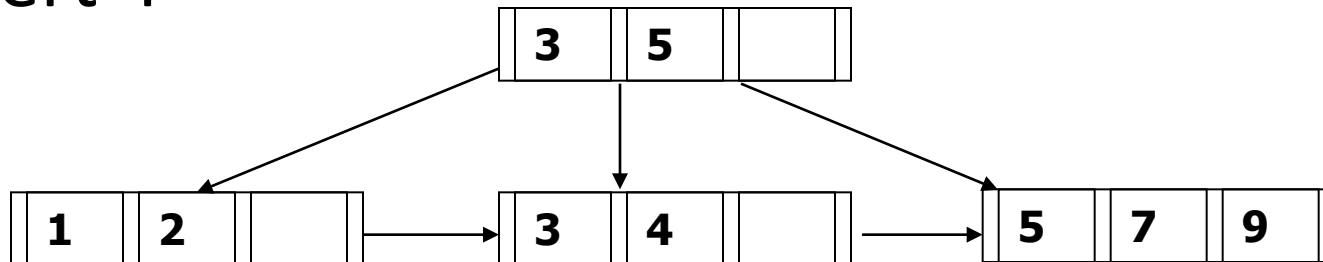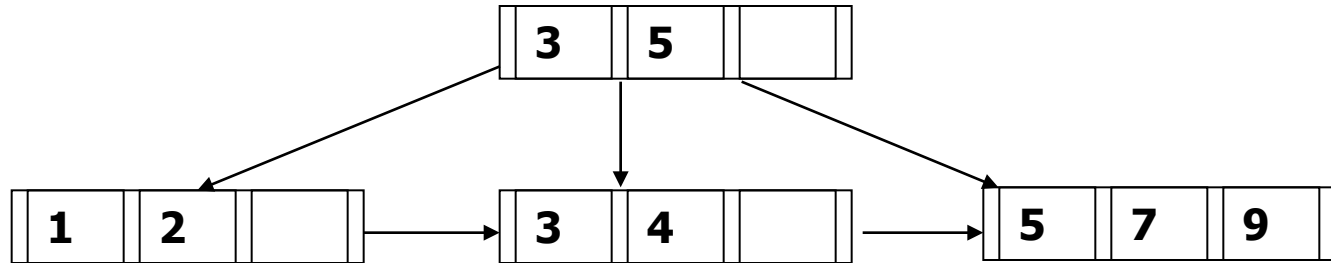

- Insert 4

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

- Insert 4

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

- Insert 6

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

3	5	

| 1 | 2 | | → | 3 | 4 | | → | 5 | 7 | 9 |

- Insert 6

3	5	7

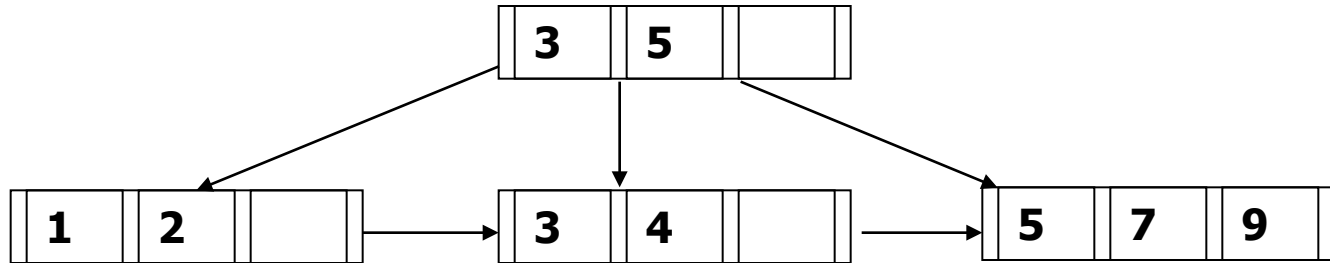| 1 | 2 | | → | 3 | 4 | | → | 5 | 6 | | → | 7 | 9 | |
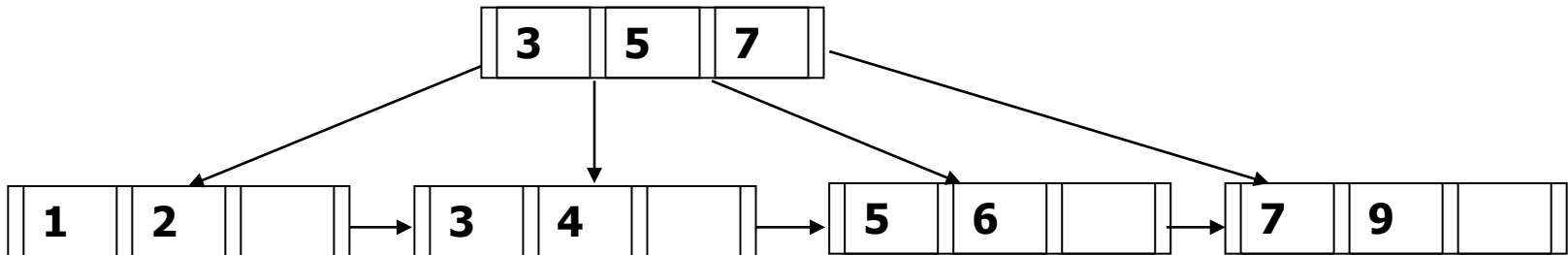
Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10
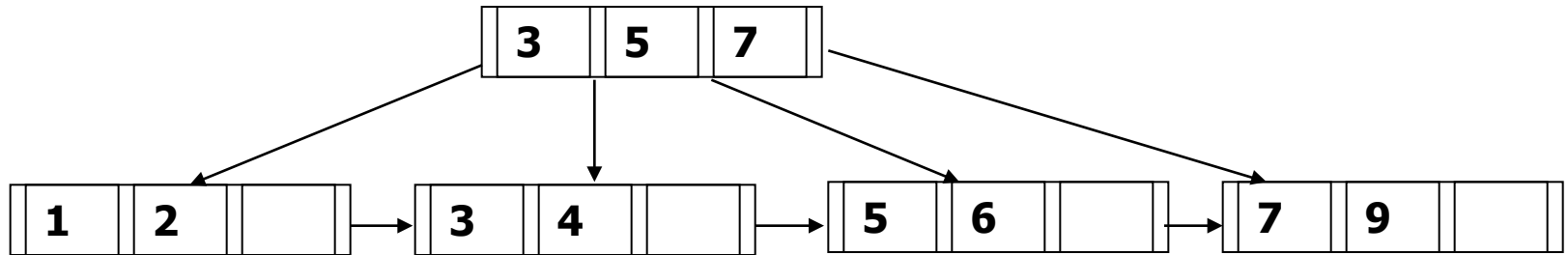
- Insert 8

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

3	5	7

| 1 | 2 | | → | 3 | 4 | | → | 5 | 6 | | → | 7 | 9 | |

- Insert 8

3	5	7

| 1 | 2 | | → | 3 | 4 | | → | 5 | 6 | | → | 7 | 8 | 9 |

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

```
            ┌───┬───┬───┐
            │ 3 │ 5 │ 7 │
            └───┴───┴───┘
```

| 1 | 2 | | | → | 3 | 4 | | | → | 5 | 6 | | | → | 7 | 8 | 9 | |
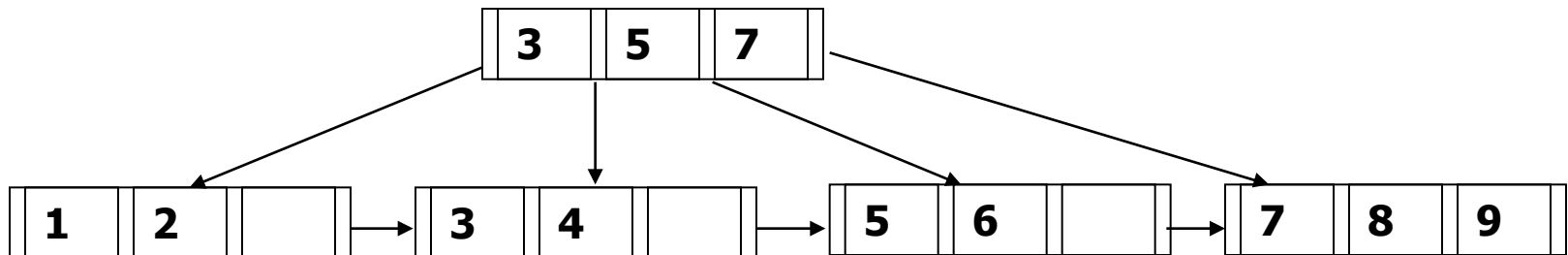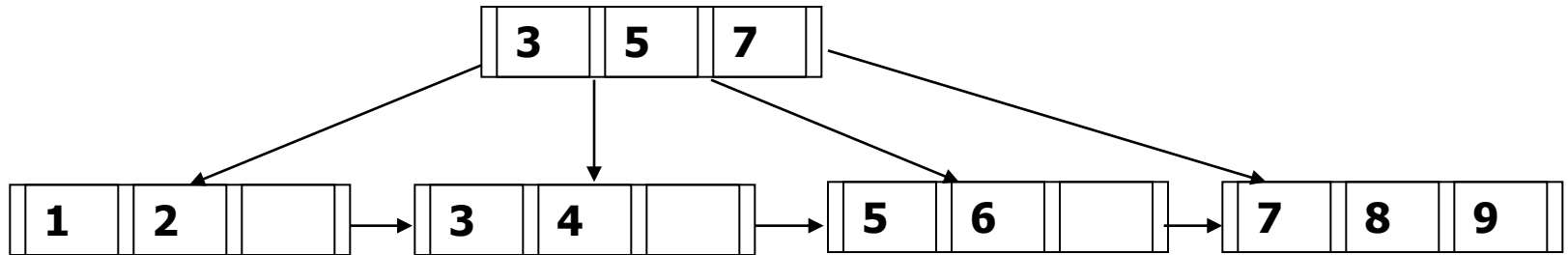
- Insert 10

Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

| 3 | 5 | 7 |

| 1 | 2 | → | 3 | 4 | → | 5 | 6 | → | 7 | 8 | 9 |

- Insert 10

| 7 | | |

| 3 | 5 | | 9 | | |

| 7 | 8 | → | 9 | 10 | |

| 1 | 2 | → | 3 | 4 | → | 5 | 6 | |
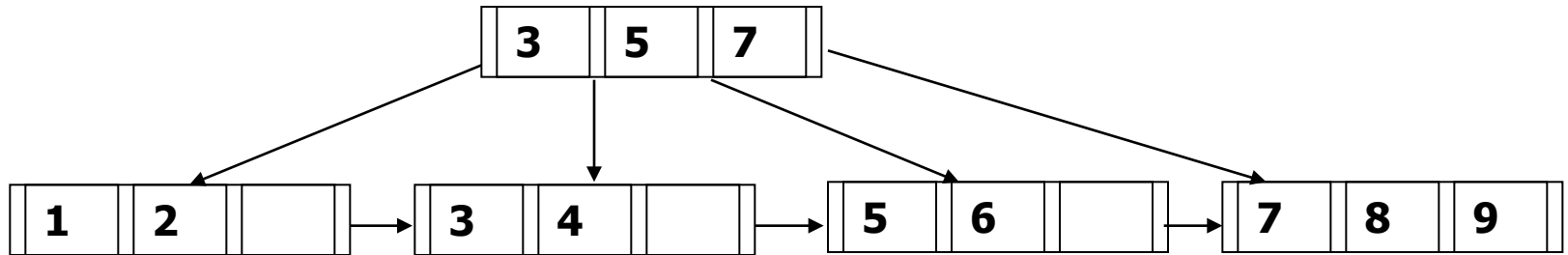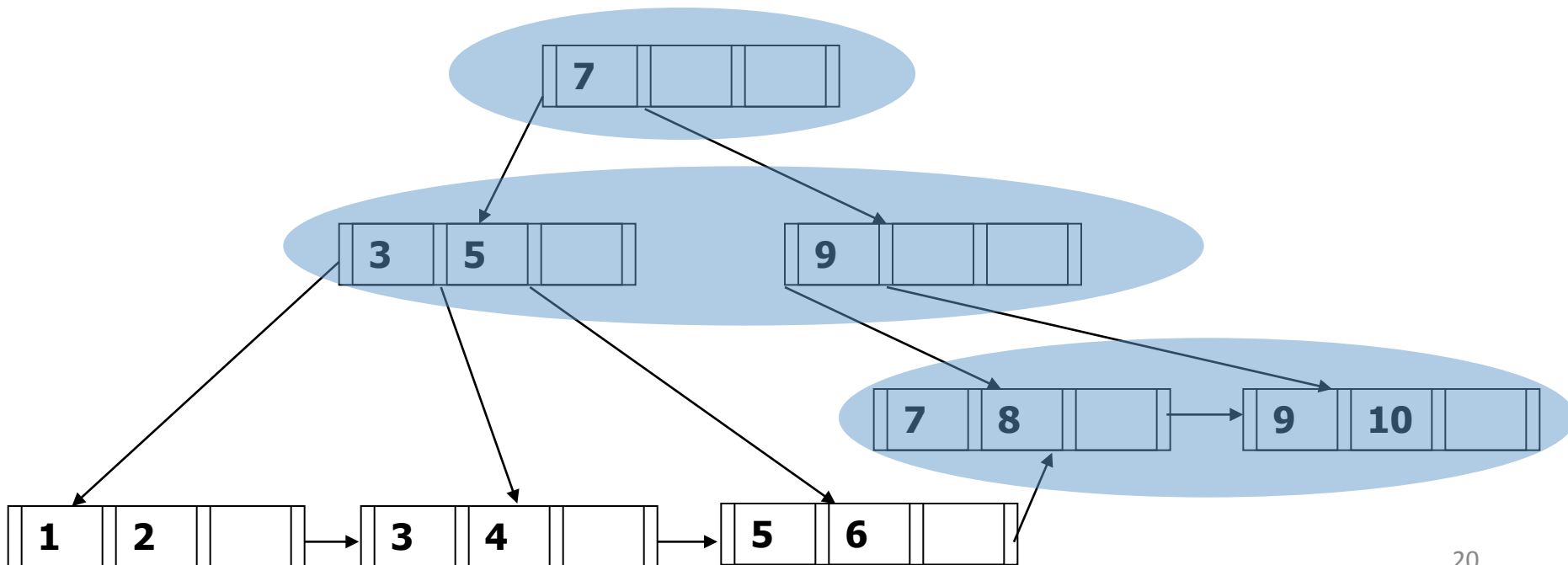
- Deletion

Show the tree after deletions
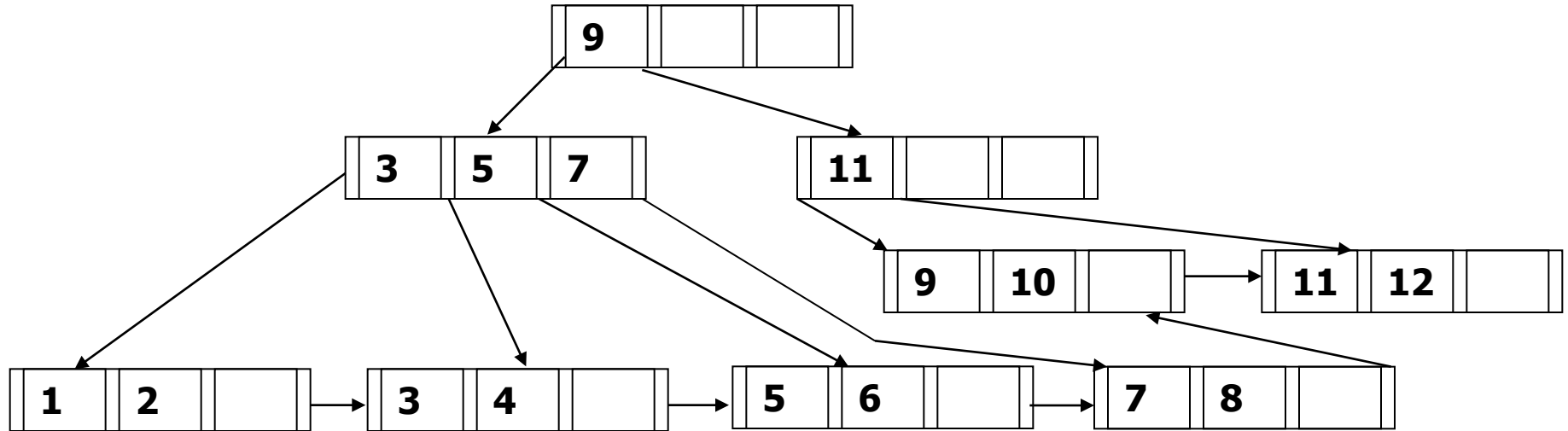
- Remove 9, 7, 8
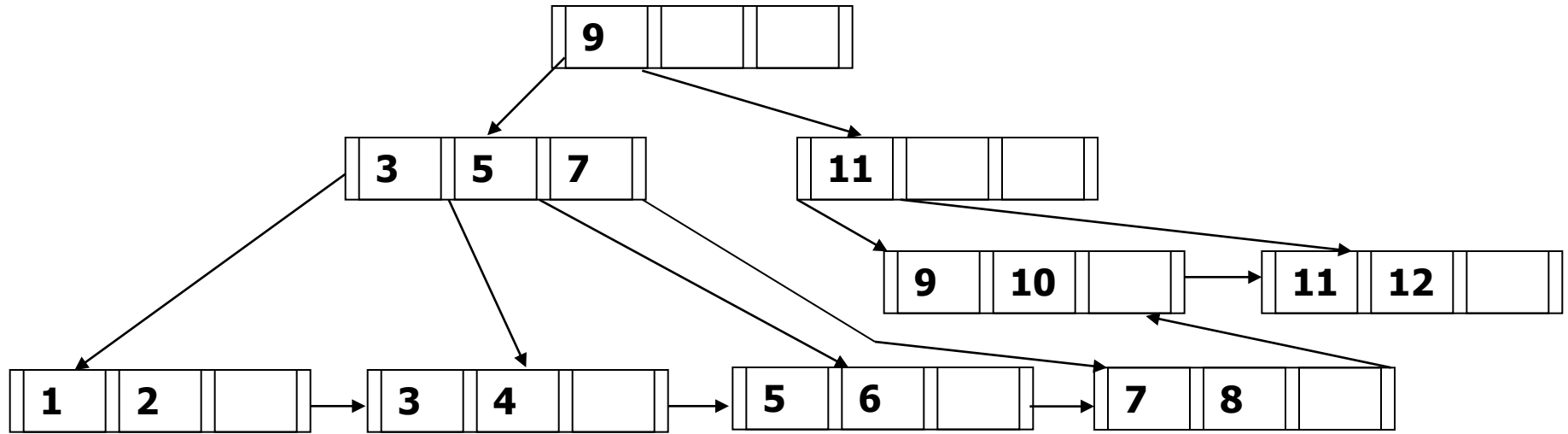
- After removing 9
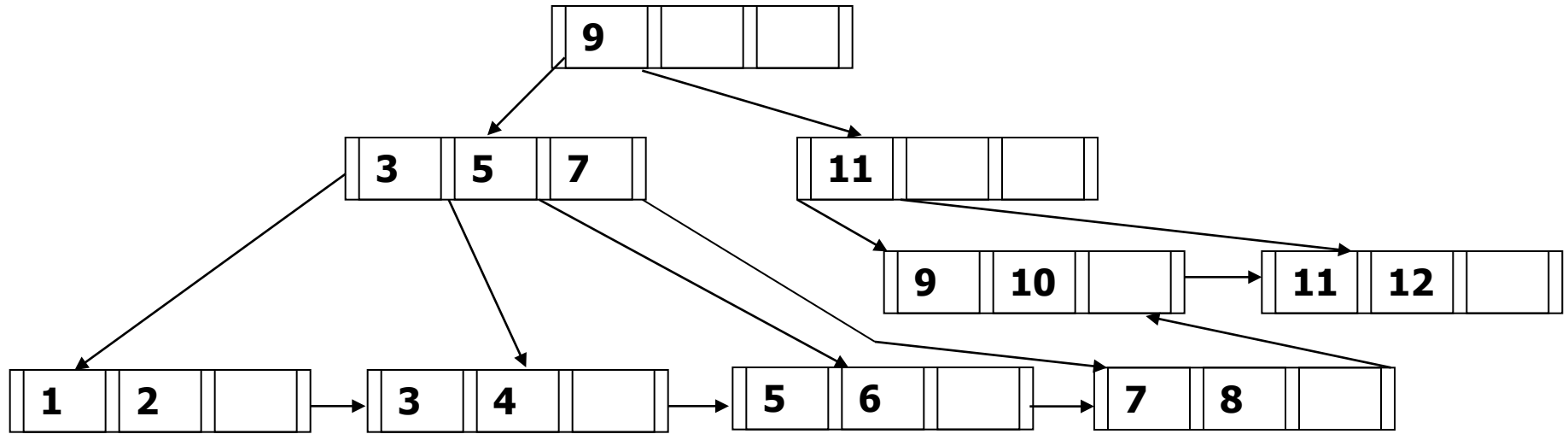
- After removing 9

Remove 9, 7, 8

```
            ┌───┬───┬───┐
            │ 7 │   │   │
            └───┴───┴───┘
           ↙           ↘
   ┌───┬───┬───┐      ┌───┬───┬───┐
   │ 3 │ 5 │   │      │ 9 │   │   │
   └───┴───┴───┘      └───┴───┴───┘
  ↙      ↓      ↘     ↙           ↘
                   ┌───┬───┬───┐      ┌───┬───┬───┐
                   │ 7 │ 8 │   │ ──→  │10 │11 │12 │
                   └───┴───┴───┘      └───┴───┴───┘
                        ↑
┌───┬───┬───┐  ┌───┬───┬───┐  ┌───┬───┬───┐
│ 1 │ 2 │   │→ │ 3 │ 4 │   │→ │ 5 │ 6 │   │
└───┴───┴───┘  └───┴───┴───┘  └───┴───┴───┘
```

- After removing 7

Remove 9, 7, 8

- After removing 7

Remove 9, 7, 8

- After removing 8

Remove 9, 7, 8

```
                              ┌───┬───┬───┐
                              │ 7 │   │   │
                              └───┴───┴───┘
                   ┌─────────┘         └─────────┐
            ┌───┬───┬───┐                  ┌───┬───┬───┐
            │ 3 │ 5 │   │                  │11 │   │   │
            └───┴───┴───┘                  └───┴───┴───┘
         ┌──┘   │   └────────┐          ┌──┘        └────────┐
         │      │            │    ┌───┬───┬───┐         ┌───┬───┬───┐
         │      │            │    │ 8 │10 │   │────────▶│11 │12 │   │
         │      │            │    └───┴───┴───┘         └───┴───┴───┘
┌───┬───┬───┐ ┌───┬───┬───┐ ┌───┬───┬───┐
│ 1 │ 2 │   │▶│ 3 │ 4 │   │▶│ 5 │ 6 │   │
└───┴───┴───┘ └───┴───┴───┘ └───┴───┴───┘
```
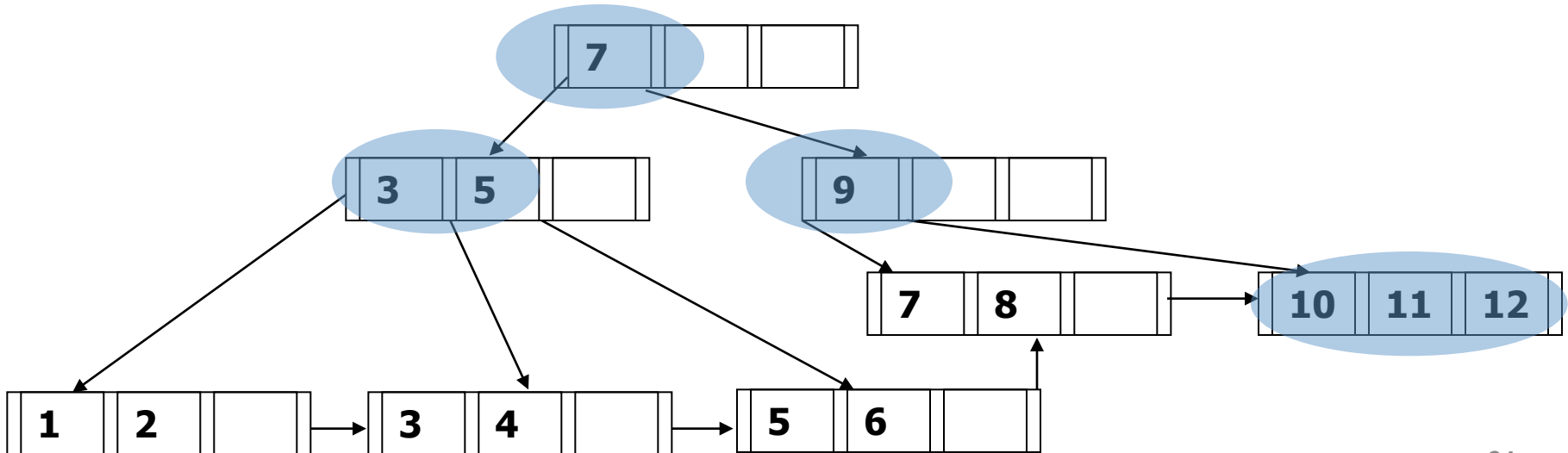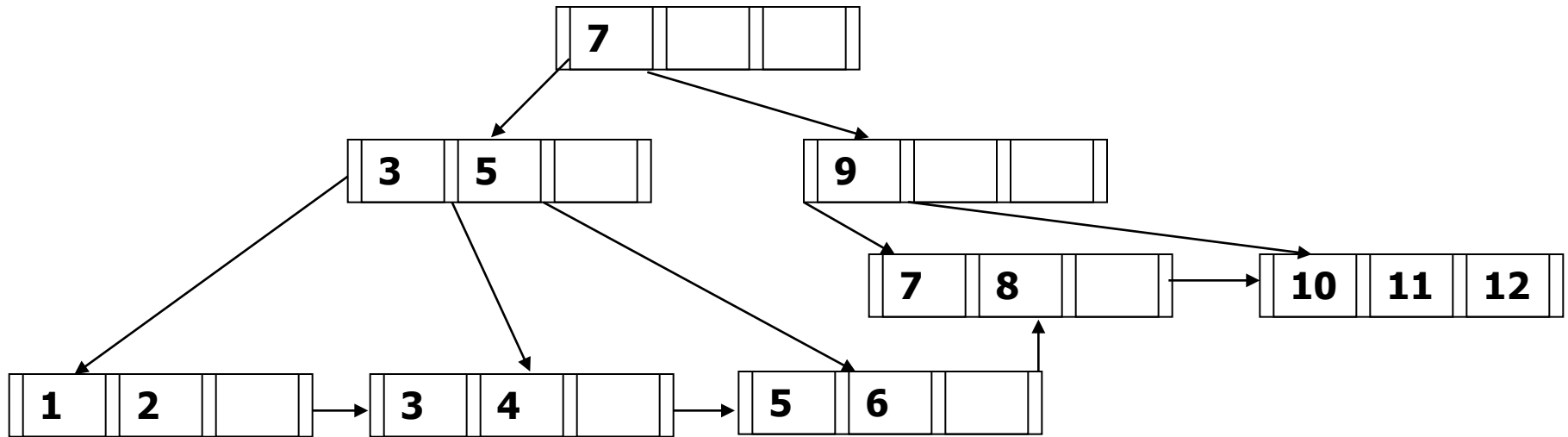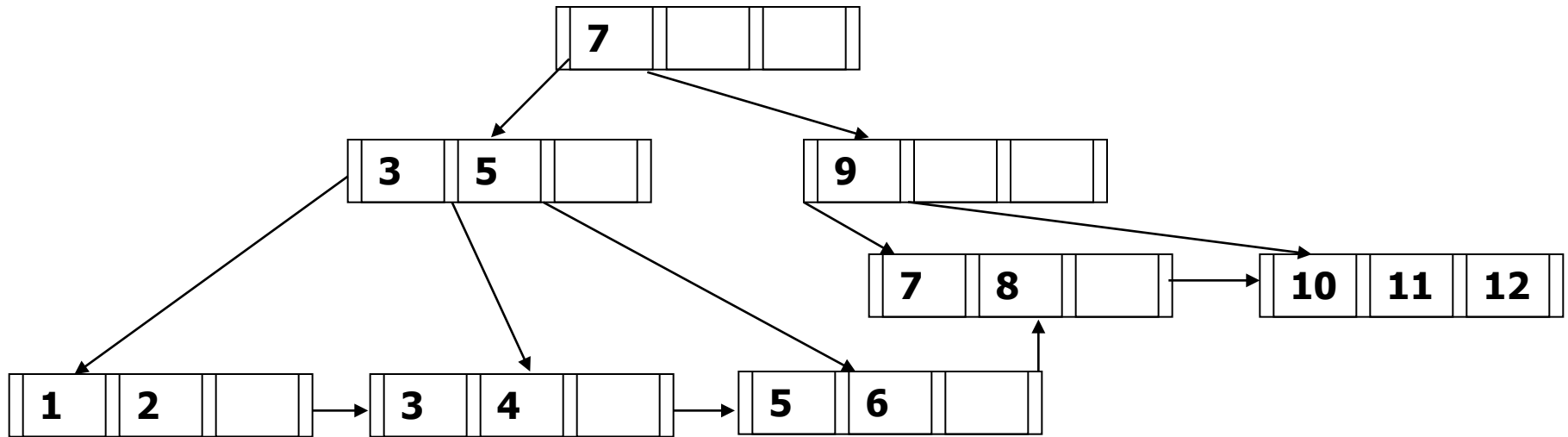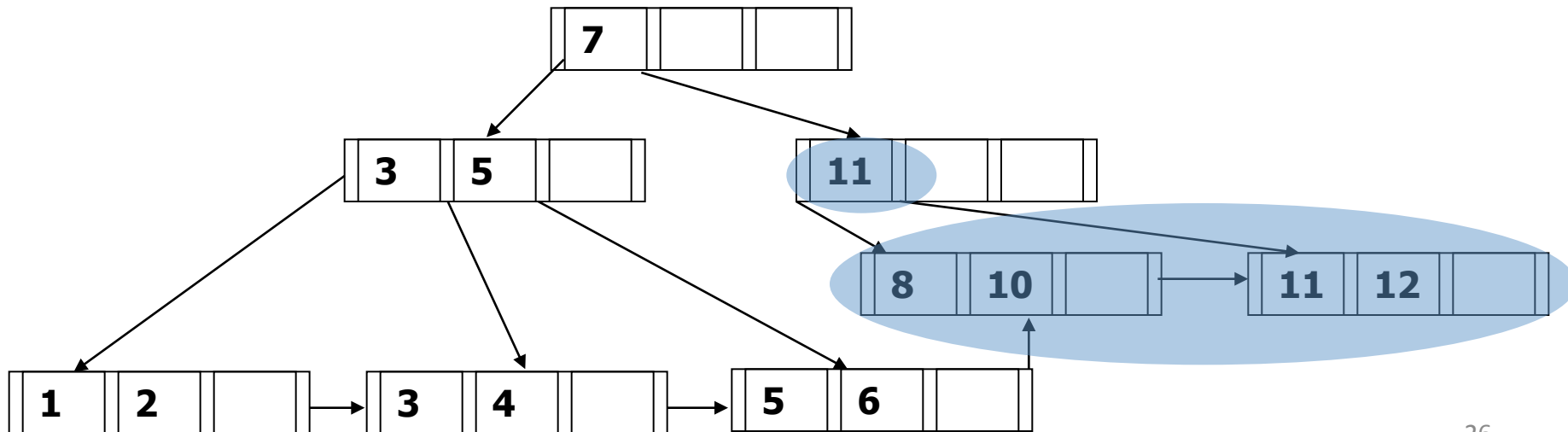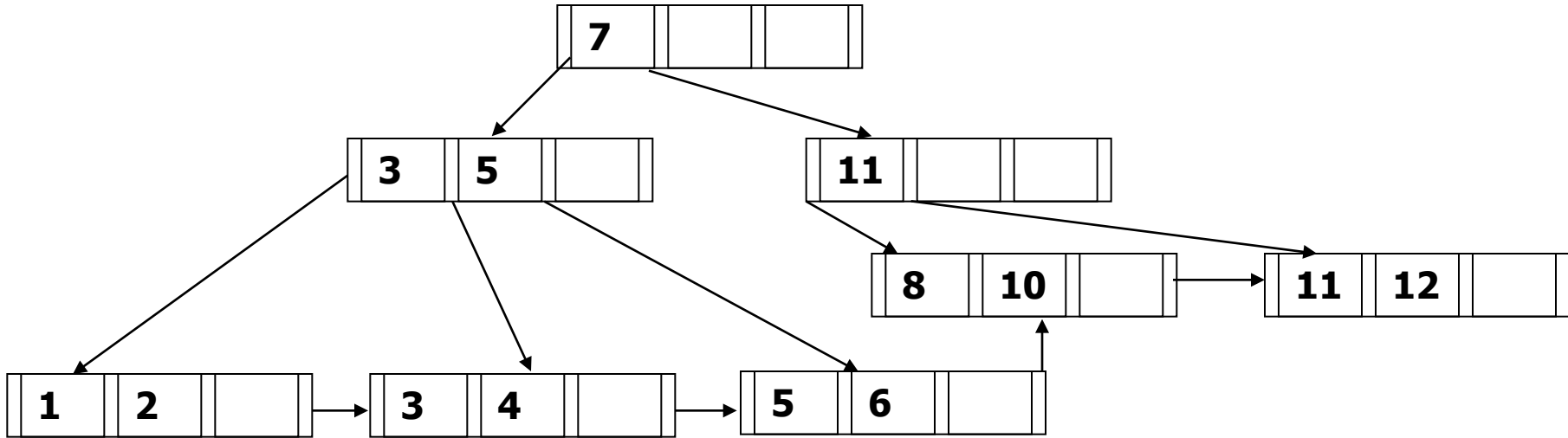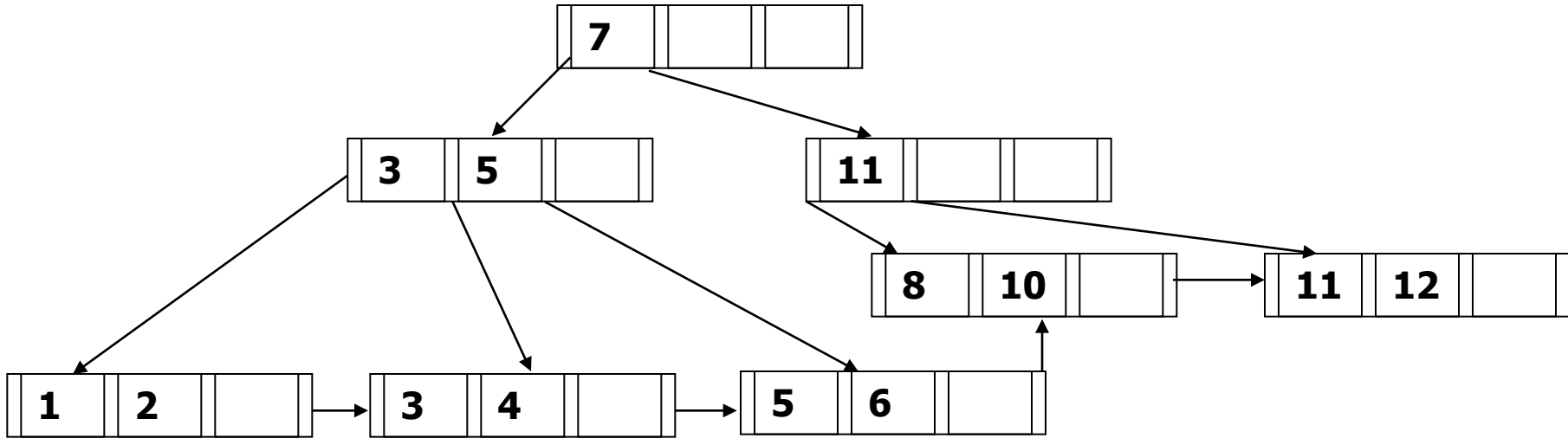
- After removing 8

```
                    ┌───┬───┬───┐
                    │ 3 │ 5 │ 7 │
                    └───┴───┴───┘
          ┌────────┘  │     │   └──────────┐
          │           │     │              │
┌───┬───┬───┐ ┌───┬───┬───┐ ┌───┬───┬───┐ ┌───┬───┬───┐
│ 1 │ 2 │   │▶│ 3 │ 4 │   │▶│ 5 │ 6 │   │─▶│10 │11 │12 │
└───┴───┴───┘ └───┴───┴───┘ └───┴───┴───┘ └───┴───┴───┘
```