Algorithms

Consider the following problem: Given a deterministic finite automata $M = \langle Q, \Sigma, \delta, q_0, F \rangle$ and an integer $n$, determine the number of strings of length exactly $n$ that are accepted by $M$.

1. **Warm-up:** Consider the DFA, $M_1$, in figure 1:

![Figure 1: DFA $M_1$](image)

List all the strings of length 3 that this automata accepts?

10 pts

2. Give a brute force algorithm to solve the general problem described above. What is the running time of this algorithm as a function $n$?

30 pts

3. Devise a polynomial-time algorithm to solve the general problem. Analyze the running-time and space complexity of your algorithm.

60 pts

**Hint:**

Let $C(q, k) = \#$ of strings of length $k$ that are accepted by $M$ if the automata is started from state $q$.

Think recursively about the function $C$. 