Qualifying Examination – Fall 1998
Automata Theory

1. Let $R_1 = \{w \in \{a,b\}^* || w| \geq 3$ and the third symbol of w is $a\}$

   (i) Give a deterministic finite automaton that accepts R_1. *5 pts*

   (ii) Consider the related language R_1',

   $R_1' = \{w \in \{a,b\}^* | \, |w| \geq 3$ and the third last symbol of w is $a\}$

   (a) Construct a minimal finite automaton (possibly nondeterministic) that accepts R_1'. *10 pts*

   (b) Construct a minimal deterministic finite automaton that accepts R_1'. *10 pts*

   You are only expected to construct the automata. You can skip proofs of their minimality.

   (iii) For a string w, let w^R denote the string that results by reversing w. For a language L define

   $L^R = \{w^R | w \in L\}$.

   Is it true that for any language L, if L is regular then L^R is also regular? Prove your answer. *15 pts*

   (iv) Is it true that for any language L, if L is accepted by a DFA with k states then L^R is accepted by some DFA with no more than k^3 states? Justify your answer. *10 pts*

2. (i) Let

   $C_1 = \{w \in \{a,b\}^* | w$ has an equal number of a's and b's $\}$.

   Is C_1 context-free? If yes, give a context-free grammar for C_1. If not, prove that this is the case. *15 pts*

   (ii) Consider the following language C_2,

   $C_2 = \{w \in \{a,b\}^* | w$ has equal number of a's and b's and the third last symbol of w is $a\}$.

   In other words, $C_2 = C_1 \cap R_1'$. Is C_2 a context-free language? Prove your answer. *25 pts*

   Hint: Think PDAs.

   (iii) Is it true that for any two languages C, R, if C is a context-free language and R is a regular language, then $C \cap R$ is a context-free language? Justify your answer. *10 pts*