Note: This quiz has five pages and five questions. Please write clear and coherent answers. You may use extra sheets for your answers if necessary.

1. Prove or disprove the following statement:
   For any two positive integer functions $f, g$, if $f(n)$ is not $O(g(n))$ then $\lg f(n)$ is not $O(\lg g(n))$.

5 pts
2. Consider the recurrence relation:

\[ T(n) = \begin{cases} 
1 & \text{if } n = 0, 1 \\
2 \times T(n - 2) + 1 & \text{otherwise}
\end{cases} \]

Use the recursion tree method to solve the above recurrence relation. \(10 \text{ pts}\)
3. Consider the following problem: Given a list of distinct integers $A$ and another integer $z$, determine if there are two different integers $x$ and $y$ in $A$ such that $x^2 + y^2 = z^2$. For example, if the list $A$ is

$$17 \ 5 \ 24 \ 14 \ 22 \ 19 \ 12$$

and $z = 13$, then $A$ does have two distinct integers such that $x^2 + y^2 = z^2$, namely, $x = 5$ and $y = 12$ ($5^2 + 12^2 = 13^2$). On the other hand if $z = 7$ there are no $x, y$ in $A$ such that $x^2 + y^2 = z^2$.

One can solve this problem by systematically computing the sum of the squares of each pair of integers $x, y$ and checking if it is equal to $z^2$. Give pseudocode for an algorithm that uses this idea to solve the problem. What is the worst-case running time of this algorithm?

10 pts

**Note:** You may assume that the list is represented as an array.
4. Give pseudocode for an algorithm that has worst-case running time $O(n \lg n)$ to solve the problem presented in question 3.

15 pts
5. Consider the following recursive function POWEROfTwo:

**Algorithm** POWEROfTwo (int n)

▷ This algorithm uses divide-and-conquer to compute $2^n$

if ($n = 0$)
  return(1)
else
  $q \leftarrow \lfloor n/2 \rfloor$
  $r \leftarrow n \mod 2$
  $x \leftarrow$ POWEROfTwo($q$)
  result $\leftarrow x \times x$
  if ($r = 0$) then
    return(result)
  else
    return(result * 2)

For an input $m$, let $T(m)$ denote the number of multiplications performed by the algorithm POWEROfTwo on input $m$.

(a) Derive a recurrence relation for $T(n)$.  

5 pts

(b) Solve the recurrence relation derived in part (a) to determine the asymptotic growth rate of $T(n)$.  

5 pts