Note: This exam has five pages. Write clear and concise answers.

1. Use the Master Method to solve the following recurrence relation:

\[
T(n) = \begin{cases} 
1 & \text{if } n = 1 \\
T(n/2) + \sqrt{n} & \text{if } n > 1 
\end{cases}
\]

5 pts
2. Consider the following recursive function **REVERSEBINARY**:

```
Algorithm REVERSEBINARY (int n)
if (n = 0)
    print(0)
else
    r ← n mod 2
    print(r)
    n ← n div 2
    if (n ≠ 0) then
        REVERSEBINARY(n)
```

Let $T(n)$ denote the running time of **REVERSEBINARY** on input $n$.

(i) Derive a recurrence relation for $T(n)$.  

5 pts

(ii) Solve the recurrence relation derived in part (i) to determine the asymptotic growth rate of $T(n)$.  

5 pts
3. (i) Define a binary search tree. 

3 pts

(ii) Give pseudocode for a procedure \texttt{RangePrint}(T, a, b) that given a pointer to the root of a binary search tree \( T \) and integers \( a \) and \( b \) with \( a < b \) prints out all the key values in \( T \) between \( a \) and \( b \) in sorted order. Analyze the run-time of your procedure.

7 pts
4. (a) Define a MAX-heap. 

5 pts

(b) Consider the MAX-heap represented by the following array $A$:

\[
\begin{array}{ccccccccccc}
i & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\end{array}
\]

Show pictorially the effect of performing first EXTRACT-MAX and then INSERT(90) on this heap.

10 pts
5. Show pictorially (including intermediate steps) the effect of carrying out the following insertions (in the order listed) in an initially empty Red-Black Tree:

\texttt{INSERT(50)}, \texttt{INSERT(40)}, \texttt{INSERT(30)}, \texttt{INSERT(20)}, \texttt{INSERT(10)}.

\textit{10 pts}