Mar 18, 2004

CS 372 Midterm Exam 50 pts

Note: This exam has five pages. Write clear and concise answers.

1. Define a binary search tree. What is the minimum height of a binary search tree that has \( n \) nodes? What is the maximum height of a binary search tree with \( n \) nodes?

5 pts
2. Give pseudocode for a procedure that given a sorted array $A[1..n]$ builds a binary search tree with key values $A[1], A[2], \ldots A[n]$ that has the minimum possible height. Your procedure must run in $O(n)$ time.

10 pts
3. Assume that you have a binary search tree $T$ where the key values are positive integers. Give pseudocode for a procedure $\text{FIND-CLOSEST}(T)$ that prints out the two key values in $T$ that are closest to each other, i.e., it should print out key values $x, y \in T$ such that $|x - y|$ is as little as possible. Analyse the worst-case running time of your algorithm.

10 pts
4. Define a Red-Black Tree.  

5 pts
5. Show pictorially (including intermediate steps) the effect of carrying out the following insertions (in the order listed) in an initially empty Red-Black Tree:

\[
\text{INSERT}(50), \text{INSERT}(40), \text{INSERT}(45), \text{INSERT}(20), \text{INSERT}(10), \text{DELETE}(20)
\]

\[10 \text{ pts}\]
6. Recall the procedure \texttt{HEAPIFY(A, i)} discussed in class that heapifies the sub-tree rooted at node \textit{i} given that the left and right sub-trees of node \textit{i} are already heaps. In class, we saw the pseudocode for the recursive version of this procedure. Give pseudocode for a non-recursive version of \texttt{HEAPIFY} that is as efficient as the recursive version.

\textit{10 pts}