Using the Archimedes Project in a Calculus 2 Class

These comments describe my experience using David Pengelley's project *Sums of numerical powers in discrete mathematics: Archimedes sums squares in the sand* in my calculus 2 class at Carthage College. Although the project was designed for use in a discrete math course, I found it worked very well, and would recommend its use.

These notes will explain the class in which I used this project, and my goals for using this project, and then walk readers through the five-day period in which I used it.

**Some background:** Carthage is a small liberal arts college. My Calculus 2 class had 25 students. The course began with integration, and covered techniques and applications of integration, and finished with a unit of finite and infinite sums.

**Motivation:** In addition to the obvious motivation (using history in class is always fun), I was hoping this project would accomplish two things. First, I wanted my students to get “hands-on” practice with the machinery of summation early. I’ve found in the past that some students are confused by the basic notation during too much of the summation unit. Second, I wanted to give them insight into the meaning of integration. The basic idea of Riemann sums can become lost by the time we finish integration techniques, and going back to this seemed a good way to increase their intuitive understanding of the process.

**Actual Implementation:** Finding five free days in a calculus class is almost impossible, but I used the lab as replacement for a big part of the ordinary lectures on summation. It replaced a lecture on Reimann Sums, and at least two lectures giving an introduction to sequences and series and techniques of summation. By the end of the project, the students have a felicity with telescoping series, changing the index of summation, calculating and bounding sums, and other tools which are helpful with more advanced topics. I used all or part of five consecutive class days to complete the project. The students worked in groups of three, and each group submitted one write-up at the end of the project. The basic timeline I followed is below (I assume the reader has a copy of this project in what follows):

**Day 1**  I began with a (very) brief introduction to summation, then let the students work in groups on problems 1 and 2. Note that problem 2 is very concerned with integral calculus, and serves as a perfect bridge for Archimedes and a modern class.

Homework for Day 1: Read pages five and six, and complete problems 3-5

**Day 2**  We began with a brief class discussion, in which I discovered almost none of my students could do problems 3-5 on their own. We carefully walked through Archimedes’s language as a class, and then the small groups resumed their work on the project at their own pace.

Homework for Day 2: Keep working; try to get through problem 10
Day 3  At this point, everyone had gotten through problem 9, though most couldn’t do number 10. Indeed, we should expect that students will struggle with this algebraically complicated problem. Because I was somewhat pressed for time, I led them through this a little more closely than I would have liked, but I still managed to leave some details for them.

Homework for Day 3: Keep working!

Day 4  I suspect that anyone teaching this project will have to modify their original plan at this point. I certainly did; we were not as far as I had hoped we would be. I decided to skip problems 18, 22, 23, and 25, and to give them a little bit of extra help on some of the other tricky problems.

Homework for Day 3: Keep working!

Day 5  By this point everyone had been thinking about higher powers, and we spent some time in full-class discussion relating what we had done to calculus. We talked briefly about finite differences, using them to understand sequence or series, and relating these ideas to integral calculus.

Homework for Day 5: As a group, carefully write up all your solutions!

Class Response: The response I got from my students was very positive. They enjoyed (with one exception) the break from routine, the idea of looking at history, and they largely expressed the belief that they got deeper insight from this than they would have from lecture. Several of them even commented positively on this project in particular in their end-of-term class evaluations.

Final thoughts: Using this project was really fun. I learned lots, but more importantly my students learned both a little bit of history, and some modern mathematics. They really did seem to be some comfortable with summation than other classes that I taught with more traditional means.