# A New Approach to Conformant Planning using CNF<sup>\*</sup>

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#### Abstract

In this paper, we develop a heuristic, progression based conformant planner, called CNF, which represents belief states by a special type of CNF formulae, called CNF-states. We define a transition function  $\Phi_{CNF}$  for computing the successor belief state resulting from the execution of an action in a belief state and prove that it is sound and complete with respect to the complete semantics defined in the literature for conformant planning. We evaluate the performance of CNF against other state-of-the-art conformant planners and identify the classes of problems where CNF is comparable with other state-of-the-art planners or scales up better than other planners. We also develop a technique called one-of relaxation which helps boost the performance of CNF. We characterize the domains where this technique can be applied and validate this idea by proposing a new set of benchmarks that is really difficult for other planners yet easy for CNF.

### **Introduction and Motivation**

Conformant planning is the planning problem in presence of incomplete information about the initial state. A conformant plan is a sequence of actions that achieves the goal from any possible initial state of the world. For example, given that the execution of an action a changes the value of a proposition f from false to true, regardless of the truth value of other propositions, the sequence containing only the action a is a conformant plan achieving f from any possible state of the world. Conformant planning is important since it removes one of the (unrealistic) assumptions about planning problems: the completeness assumption which states that agent has complete information about the initial state.

Since the problem is introduced (Smith and Weld 1998), several conformant planners have been developed. To date, the most efficient conformant planners are Conformant-FF (CFF) (Brafman and Hoffmann 2004), KACMBP (Cimatti, Roveri, and Bertoli 2004), POND (Bryce, Kambhampati, and Smith 2006),  $\pm 0$  (Palacios and Geffner 2007), CPA (Tran et al. 2009), and DNF (To, Pontelli, and Son 2009). One of the major difficulties in the building of a conformant planner lies in dealing with the uncertainty about the initial state, which is often represented by a set of formulas describing the possible initial states of the world, which is often referred as the *initial belief state*. In a domain with n propositions, the size of the initial belief state can be  $2^n$ .

The representation of belief states affects the performance of a conformant planner in two ways. First, it can quickly increase the memory usage of the planner, leading to the undesirable "Out of memory" situation, if the size of the belief state is large. Second, it directly influences the time complexity in computing the successor belief states.

Indeed, the literature is rich in methods for representing of and dealing with belief states in conformant planners. CFF does not have a direct encoding of the belief states in its search process. Rather, it stores the sequence of actions reaching to the belief states and recomputes the belief states whenever it is necessary.<sup>1</sup> Checking whether an action is executable needs a call to a SAT-solver. POND uses ordered binary decision diagram (OBDD) (Bryant 1992) in its representation and relies on the OBDD-library to compute the successor belief states. A disadvantage of this approach is that the size of the OBDD can be very large-the structure of the OBDD is sensitive to the ordering of variables and the manipulation of the OBDD might require intermediate OB-DDs of exponential size. t0 translates a conformant planning problem to a classical planning problem, uses a classical planner (FF) to solve the new problem, and then converts the solutions of the new problem to solutions of the original problem. The size of the initial state (the number of the fluents) in the new problem, however, could be exponential to the size of the original problem if completeness is required. CPA identifies and keeps only a set of approximation states, which guarantees completeness of the planner, in its search for solution; thus reducing the size of the belief states, in many cases, significantly. The price CPA has to pay is that the overhead for computing the initial approximation state is sometimes quite significant, leading to its inability to start the search process. DNF goes to an extreme by employing an explicit representation of belief states using disjunctive normal form. DNF expands the DNF representation on demand. As such, it can avoid some problems faced by CPA, CFF, or t0 and can solve more problems than these planners (To, Pontelli, and Son 2009).

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<sup>&</sup>lt;sup>1</sup>CFF does implement some techniques which help reduce the number of recomputations.

It is worth mentioning that regardless of the representation of the belief states, *all planners* have difficulties scaling up when the size of the initial belief state is large. For example, for the coins-21 instance, from the IPC-2006, all planners fail to find a solution either because of "out of memory" error or because of "time out." The initial belief state of this problem contains  $10^{16}$  states, comparing to the "easy instances" in the same domain such as coins-20, whose initial belief state contains less than  $10^6$  possible states, and can be solved by all planners in less than two minutes.

The above discussion motivates us to investigate alternative representation of belief states in conformant planning. More precisely, we develop a conformant planner, called CNF, that uses a specific form of CNF, called *reduced CNF*, to represent belief states. We compare CNF with other planners and our experiments show that the new representation does incur some overhead which cannot be easily overcame. In order to exploit the new representation, we develop a new technique called one-of relaxation and implement it in CNF. This technique helps further reducing the size of the initial belief state, allowing the planner to solve instances which cannot be solved by other planners due to the size of the initial belief state. We provide a characterization of domains in which this technique can be applied and evaluate this idea by proposing a new set of benchmarks that is hard for the other planners yet easy for CNF.

# **Background: Conformant Planning**

A planning problem is a tuple  $P = \langle F, O, I, G \rangle$ , where F is a set of propositions, O is a set of actions, I describes the initial state of the world, and G describes the goal. A *literal* is either a proposition  $p \in F$  or its negation  $\neg p$ .  $\overline{\ell}$  denotes the complement of a literal  $\ell$ —i.e.,  $\overline{\ell} = \neg \ell$ , where  $\neg \neg p = p$  for  $p \in F$ . For a set of literals  $L, \overline{L} = \{\overline{\ell} \mid \ell \in L\}$ . A conjunction of literals is often viewed as the set of its literals.

A set of literals X is *consistent* if there exists no  $p \in F$ such that  $\{p, \neg p\} \subseteq X$ . A set of literals X is *complete* if for each  $p \in F$ ,  $\{p, \neg p\} \cap X \neq \emptyset$ . A *state* s is a consistent and *complete* set of literals. A *belief state* is a set of states. We will often use lowercase (resp. uppercase) letter, possibly with indices, to represent a state (resp. a belief state).

Each action a in O is associated with a precondition  $\phi$  (denoted by pre(a)) and a set of conditional effects of the form  $\psi \to \ell$  (also denoted by  $a : \psi \to \ell$ ), where  $\phi$  and  $\psi$  are sets of literals and  $\ell$  is a literal.

A state s satisfies a literal  $\ell$ , denoted by  $s \models \ell$ , if  $\ell \in s$ . s satisfies a conjunction of literals X, denoted by  $s \models X$ , if it satisfies every literal belonging to X. The satisfaction of a formula in a state is defined in the usual way. Likewise, a belief state S satisfies a literal  $\ell$ , denoted by  $S \models \ell$ , if  $s \models \ell$  for every  $s \in S$ . S satisfies a conjunction of literals X, denoted by  $S \models X$ , if  $s \models X$  for every  $s \in S$ .

Given a state s, an action a is executable in s if  $s \models pre(a)$ . The effect of executing a in s is

$$e(a,s) = \{\ell \mid \exists (a:\psi \to \ell). \ s \models \psi\}$$

The transition function, denoted by  $\Phi$ , in the planning domain of P is defined by

$$\Phi(a,s) = \begin{cases} s \setminus \overline{e(a,s)} \cup e(a,s) & s \models pre(a) \\ \bot & \text{otherwise} \end{cases}$$
(1)

where  $\perp$  denotes a fail state.

 $\Phi$  is extended to define  $\Phi$ , a transition function which maps sequences of actions and belief states to belief states for reasoning about the effects of plans. Let S be a belief state. We say that an action a is executable in a belief state S if it is executable in every state belonging to S. Let  $\alpha_n = [a_1, \ldots, a_n]$  be a sequence of actions and  $\alpha_i = [a_1, \ldots, a_i]$ :

• If n = 0 then  $\widehat{\Phi}([], S) = S$ ;

- if  $\widehat{\Phi}(\alpha_{n-1}, S) = \bot$  or  $a_n$  is not executable in  $\widehat{\Phi}(\alpha_{n-1}, S)$ , then  $\widehat{\Phi}(\alpha_n, S) = \bot$ ;
- if  $\widehat{\Phi}(\alpha_{n-1}, S) \neq \bot$  and  $a_n$  is executable in  $\widehat{\Phi}(\alpha_{n-1}, S)$ then  $\widehat{\Phi}(\alpha_n, S) = \{ \Phi(a_n, s') \mid s' \in \widehat{\Phi}(\alpha_{n-1}, S) \}.$

The initial state of the world I is a belief state and is represented by a formula. In all benchmarks, I is a conjunction of a set of literals, a set of one-of statements (resp. orstatements)—representing an exclusive-or (resp. logical or) of its components. By  $S_I$  we denote the set of all states satisfying I. Typically, the goal description G can contain literals and or-clauses.

A sequence of actions  $\alpha = [a_1, \ldots, a_n]$  is a solution of P if  $\widehat{\Phi}(\alpha, S_I)$  satisfies G. In this paper, for an action a, we will denote with  $C_a$  the set of conditional effects of a.

# **Representing Belief States by CNF Formulae**

In this section we develop the theoretical basis for the development of a conformant planner which uses a specific type of CNF formulae, which will be introduced shortly, to represent belief states. Our first task is to define a progression function  $\Phi_{CNF}$ , similar to the function  $\Phi$ , for computing the successor belief state.

A *clause* is a set of fluent literals. A clause is *trivial* if it contains the set  $\{f, \neg f\}$  for some fluent f. A *CNF formula* is a set of clauses. A literal  $\ell$  is in a CNF formula  $\varphi$ , denoted by  $\ell \in \varphi$ , if there exists a clause  $\alpha \in \varphi$  such that  $\ell \in \alpha$ . A *unit clause* is a singleton set, i.e. it contains only one literal. The literal in a unit clause is called *unit literal*. A clause  $\alpha \in \varphi$ . A clause a clause  $\beta$  (or  $\beta$  is *subsumed by*  $\alpha$ ) if  $\alpha \subset \beta$ . A clause is subsumed by a CNF formula if it is subsumed by some clause in the CNF formula. Obviously, if  $\varphi'$  is obtained by removing some subsumed clause(s) in a CNF formula  $\varphi$  then  $\varphi' \equiv \varphi$ .

**Definition 1.** A CNF-formula  $\varphi$  is called a CNF-state if

- $\varphi$  does not contain a trivial clause;
- $\varphi$  does not contain two clauses  $\gamma$  and  $\delta$  such that  $\gamma$  subsumes  $\delta$ ; and
- for every unit clause {ℓ} in φ, ℓ ∉ φ.
   A set of CNF-states is called a CNF-belief state.

Intuitively, a CNF-state  $\varphi$  is minimal in the sense that it does not contain redundant clauses and can be viewed as a set of states. For example, if  $F = \{p,q\}$ , the CNFstate  $\varphi_1 = \{p\}$  encodes the set of states in which p is true, i.e., the set  $\{\{p,q\}, \{p,\neg q\}\}$ ; the CNF-state  $\varphi_2 = \{\neg p\}$ encodes the set of states in which p is false; the set of CNF-states  $\{\varphi_1, \varphi_2\}$  represents all possible states of the domain. Observe also that even though  $\varphi_1$  is equivalent to  $\varphi = \{\{p\}, \{p,q\}\}, \varphi$  is not a CNF-state as it violates the second condition of Def. 1. Furthermore, observe that a CNF-formula might be represented by several CNF-states. For example, the formula *p* can be represented by either  $\{p\}$  or  $\{\{p,q\}, \{p,\neg q\}\}$ .

It is easy to see that an arbitrary CNF-formula can be converted into an equivalent CNF-state and there exist different algorithms for this purpose. In CNF, we implemented a fixed-point algorithm which is a modification of an effective CNF-preprocessor using unit-propagation and subsumption techniques. The algorithm is deterministic and runs in polynomial time in the size of the formula. Furthermore, it yields a CNF-state that cannot be further simplified. For our discussion, it suffices to note that  $r(\varphi)$ , the result of the algorithm, is a CNF-state equivalent to  $\varphi$ .

A literal  $\ell$  (a set of literals  $\gamma$ ) is true in a CNF-state  $\varphi$  if  $\varphi \models \ell$  ( $\varphi \models \ell$  for every  $\ell \in \gamma$ ), where  $\models$  denotes the standard entailment relation. Also, by  $\varphi_{\ell}$  (resp.  $\varphi_{\bar{\ell}}$ ) we denote the set of clauses in  $\varphi$  which contain  $\ell$  (resp.  $\bar{\ell}$ ).

We define some operations for the manipulation of CNF formulae, which will be used in manipulating CNF-states.

**Definition 2.** Let  $\varphi$  be a CNF formula and  $\ell$  be a literal. By  $\varphi - \ell$  we denote the CNF formula obtained by removing from  $\varphi$  every occurrence of  $\ell$ .

E.g., 
$$p \land (q \lor \neg r) - q = p \land \neg r$$
 and  $p \land (q \lor \neg r) - p = q \lor \neg r$ .

**Definition 3.** For two CNF formulae  $\varphi = \{\alpha_1, \ldots, \alpha_n\}$  and  $\psi = \{\beta_1, \ldots, \beta_m\}$ , the cross-product of  $\varphi$  and  $\psi$ , denoted by  $\varphi \times \psi$ , is the CNF-formula defined by  $\{\alpha_i \cup \beta_j \mid \alpha_i \in \varphi, \beta_j \in \psi\}$ . If either  $\varphi$  or  $\psi$  is empty then  $\varphi \times \psi = \emptyset$ .

Observe that  $\varphi \times \psi$  is nothing but a transformation of  $\varphi \lor \psi$  to a CNF formula. For a set of CNF formulae  $\Psi = \{\varphi_1, \ldots, \varphi_n\}, \times [\Psi]$  denotes  $\varphi_1 \times \varphi_2 \times \ldots \times \varphi_n$ , a CNF-formula equivalent to  $\bigvee_{i=1}^n \varphi_i$ .

Given a CNF-state  $\varphi$ , we define the function Update which encodes the CNF-state after the execution of an action, that causes a literal  $\ell$  to be true, in  $\varphi$  as follows.

**Definition 4.** Let  $\varphi$  be a CNF-state and  $\ell$  a literal. The update of  $\varphi$  by  $\ell$ , denoted by Update $(\varphi, \ell)$ , is defined by:

• If  $\ell$  is a unit clause in  $\varphi$  then  $Update(\varphi, \ell) = \varphi$ .

• If  $\overline{\ell}$  is a unit clause in  $\varphi$ , then  $Update(\varphi, \ell) = (\varphi - \overline{\ell}) \wedge \ell$ .

$$Update(\varphi, \ell) = r((\varphi \setminus (\varphi_{\ell} \cup \varphi_{\bar{\ell}})) \land \ell \land (\varphi_{\ell} - \ell) \times (\varphi_{\bar{\ell}} - \bar{\ell}))$$

Intuitively,  $Update(\varphi, \ell)$  encodes Equation (1) when the set of effects is  $\{\ell\}$  and the action is executable. As  $\varphi$  represents a belief state  $S = \{s \mid s \models \varphi\}$ ,  $Update(\varphi, \ell)$  should be equivalent to the formula  $\bigvee_{s \in S} (s \setminus \{\ell\} \cup \{\ell\})$ . As such, the first two cases are obvious. For the third case, observe that if  $\varphi$  is equivalent to  $\varphi' = (\varphi_0 \land (\varphi_\ell - \ell) \land (\varphi_{\bar{\ell}} - \ell)) \lor (\varphi_0 \land (\varphi_{\bar{\ell}} - \ell) \land \ell)$ , where  $\varphi_0 = \varphi \setminus (\varphi_\ell \cup \varphi_{\bar{\ell}})$ . The first disjunct does not contain  $\ell$  or  $\bar{\ell}$ , the second contains  $\bar{\ell}$ , and the third contains  $\ell$ . The last two disjuncts can be updated according to the first two cases. Updating the first disjunct is done by adding the unit-clause  $\{\ell\}$  to it. The formula in the definition is obtained by a simplification of the obtained formula. The definition is illustrated in the next few examples:

- $\circ \ Update(\{\{f\}, \{g, \neg p\}\}, p) = \{\{f\}, \{p\}\}\$
- $\circ Update(\{\{f\}, \{h, p\}\}, p) = \{\{f\}, \{p\}\}\$
- $Update(\{\{f\}, \{g, \neg p\}, \{h, p\}\}, p) = \{\{f\}, \{p\}, \{h, g\}\}$ We can prove the following proposition.<sup>2</sup>

**Proposition 1.** Given a CNF-state  $\varphi$  and two literals  $\ell_1$  and  $\ell_2$  such that  $\ell_1 \neq \bar{\ell}_2$ , then  $Update(Update(\varphi, \ell_1), \ell_2) = Update(Update(\varphi, \ell_2), \ell_1)$ .

Proposition 1 shows that the result of updating a CNFstate  $\varphi$  using a consistent set of literals L is independent from the order in which the various literals of Lare introduced. Given a consistent set of literals L, we denote  $Update(\varphi, \emptyset) = \varphi$  and  $Update(\varphi, L) =$  $Update(Update(\varphi, \ell), L \setminus \{\ell\})$  for any  $\ell \in L$  if  $L \neq \emptyset$ .

Our goal is now to define the transition function  $\Phi_{CNF}$ . Given an action a with the precondition pre(a) and its set of conditional effect  $C_a$  and a CNF-state  $\varphi$ , we need to define the successor CNF-state  $\Phi_{CNF}(a,\varphi)$ . Since  $\varphi$  is a CNF-state, when computing  $\Phi_{CNF}(a,\varphi)$ , for each  $\psi \to \ell$  in  $C_a$ , there are three cases that need to be considered: (i)  $\varphi \models \psi$ ; (ii)  $\varphi \models \neg \psi$ ; and (iii)  $\varphi \not\models \psi$  and  $\varphi \not\models \neg \psi$ . As an example, if  $\varphi = p \land q$  and  $\psi = r$ , then we have  $\varphi \not\models \psi$  and  $\varphi \not\models \neg \psi$ . As such, to define  $\Phi_{CNF}$ , we need the following definition.

**Definition 5.** Let  $\varphi$  be a CNF formula and  $\gamma$  a consistent set of literals. The enabling form of  $\varphi$  w.r.t.  $\gamma$ , denoted by  $\varphi + \gamma$ , is a set of CNF formulae and is defined as

$$\varphi + \gamma = \begin{cases} \{r(\varphi)\} & \text{if } \varphi \models \gamma \text{ or } \varphi \models \neg \gamma \\ \{r(\varphi \land \gamma), r(\varphi \land \neg \gamma)\} & \text{otherwise} \end{cases}$$

where  $\neg \gamma$  is the clause  $\overline{\gamma} = \{\overline{l} | l \in \gamma\}, \varphi \land \neg \gamma = \varphi \cup \{\overline{\gamma}\}, and \varphi \land \gamma = \varphi \cup \{\{l\} | l \in \gamma\}$ 

It is easy to see that  $\varphi + \gamma$  is a set of CNF formulae such that for every  $\delta \in \varphi + \gamma$ ,  $\delta \models \gamma$  or  $\delta \models \neg \gamma$ . For a set of CNF formulae  $\Psi$ , let  $\Psi + \gamma = \bigcup_{\varphi \in \Psi} (\varphi + \gamma)$ . It holds that

**Proposition 2.** Let  $\varphi$  (resp.  $\Psi$ ) be a CNF formula (resp. set of CNF formulae). If  $\gamma$  is a consistent set of literals, then  $\varphi + \gamma$  (resp.  $\Psi + \gamma$ ) is a CNF-belief state equivalent to  $\varphi$ (resp.  $\Psi$ ). If  $\gamma_1$  and  $\gamma_2$  are two consistent sets of literals then  $(\varphi + \gamma_1) + \gamma_2 \equiv (\varphi + \gamma_2) + \gamma_1$ .

**Definition 6.** Let a be an action with the set of conditional effects  $C_a$ . A CNF formula  $\varphi$  is called enabling for a if for every conditional effect  $\psi \rightarrow \ell$  in  $C_a$ , either  $\varphi \models \psi$  or  $\varphi \models \neg \psi$  holds. A set of CNF formulae  $\Psi$  is enabling for a if every CNF formula in  $\Psi$  is enabling for a.

For an action a and a CNF-state  $\varphi$ , let  $enb_a(\varphi) = ((\varphi + \psi_1) + \ldots) + \psi_k$  where  $C_a = \{\psi_1 \to \ell_1, \ldots, \psi_k \to \ell_k\}.$ 

**Proposition 3.** For every CNF-state  $\varphi$  and action a,  $enb_a(\varphi)$  is a CNF-belief state equivalent to  $\varphi$  and enabling for a.

For an action a and a CNF-state  $\varphi$ , the effect of a in  $\varphi$ , denoted  $e(a, \varphi)$ , is defined as follows.  $e(a, \varphi) = \{\ell | \psi \rightarrow \ell \in C_a, \varphi \models \psi\}$ . We are now ready to define the function  $\Phi_{CNF}$ .

<sup>&</sup>lt;sup>2</sup>Due to lack of space, all proofs are omitted. They can be found at www.cs.nmsu.edu/~sto/full\_cnf\_paper.pdf.

**Definition 7.** Let  $\varphi$  be a CNF-state and let a be an action. By  $\Phi_{CNF}(a, \varphi)$  we denote the transition function for CNF-states:

- $\Phi_{CNF}(a,\varphi) = r(\times [\{Update(\phi, e(a, \phi)) | \phi \in enb_a(\varphi)\}])$ if  $\varphi \models pre(a)$ ; and
- $\Phi_{CNF}(a,\varphi) = \bot$  otherwise.
- The next lemma proves that  $\Phi_{CNF}$  is equivalent to  $\Phi$ .

**Lemma 1.** Let  $\phi$  be a CNF-state enabling an action a. It holds that  $Update(\phi, e(a, \phi)) \equiv \{s \setminus \overline{e(a, s)} \cup e(a, s) | s \in BS(\phi)\}$  where  $BS(\varphi)$  denotes the belief state consisting of states satisfying  $\varphi$ .

Let  $\Phi_{CNF}$  is the function defined by extending  $\Phi_{CNF}$  to allow for reasoning about the effects of an action sequence in the same manner as extending  $\Phi$  to  $\widehat{\Phi}$ . Using Lemma 1, one can prove the correctness of the following theorem.

**Theorem 1.** For every CNF state  $\varphi$  and action sequence  $\alpha_n$ ,  $\widehat{\Phi_{CNF}}(\alpha_n, \varphi) \equiv \widehat{\Phi}(\alpha_n, BS(\varphi))$ .

The above theorem shows that  $\widehat{\Phi_{CNF}}$  is equivalent to the complete semantics defined by  $\widehat{\Phi}$ . Thus, any planner using  $\widehat{\Phi_{CNF}}$  in its search for solutions will be sound and complete.

### **CNF-States and one-of Statements**

One of the main methods used in specifying uncertainty in conformant planning benchmarks is the use of the one-of construct. This construct provides a way to specify mutual exclusive information about certain proposition (e.g., a truck t is at one (and only one) place  $p_1, p_2, p_3$  is expressed by the statement one-of  $(at(t, p_1), at(t, p_2), at(t, p_3))$ . The size of the initial belief state (the number of states) depends largely on the number of one-of statements in the problem specification. The following proposition discusses this issue.

**Proposition 4.** A one-of statement c can be converted to

- an equivalent DNF formula of minimum size of  $|c|^2$ ; and
- an equivalent CNF formula of minimum size of  $\Theta(|c|^2)$ .
- where |c| denotes the number of literals in c.

A set  $S = \{c_1, \ldots, c_n\}$  of independent one-of statements can be converted to an equivalent DNF formula of minimum size  $\prod_{i=1}^{n} |c_i|^2$  and to an equivalent CNF formula of minimum size  $\Theta(\sum_{i=1}^{n} |c_i|^2)$ .

In the above proposition we use *independent* which will be defined later in the Definition 9. This proposition shows that the representation of belief state affect the size of the initial belief state, which the planner needs to deal with. This is also the main reason behind the development of the one-of combination technique by CPA which greatly influences the performance of CPA and DNF. This motivates us to develop an alternative technique to the one-of combination technique for use in the CNF planner.

Intuitively, the CNF-representation favors the second way of specifying uncertainty in conformant planning benchmarks, the use of or statements. Can we replace one-of statements by or statements? Surprisingly, in many benchmarks, the answer for this question is yes. We call this technique as one-of *relaxation*. To illustrate this technique, let us consider the dispose(m, n) problem, one of the benchmarks in IPC-2008. The goal of this problem is to retrieve *m* objects  $o_1, \ldots, o_m$ , whose initial locations are unknown, and placing them in a trash can. The initial information is specified by  $\delta_i = \text{one-of}(at(o_i, p_1), \ldots, at(o_i, p_n))$ . We say that  $\delta'_i = \text{or}(at(o_i, p_1), \ldots, at(o_i, p_n))$  is a relaxation of the one-of statement  $\delta_i$  since the belief state specified by the or statement contains all states belonging to the belief state specified by the one-of statement. Let the planning problem P' be obtained from the problem dispose(m, n)by replacing  $\delta_i$  with  $\delta'_i$  for every *i*. Obviously, the size of the CNF-state representing the initial state of dispose(m, n) is much larger than the size of the CNF-state encoding the initial state of P' ( $m \times \Theta(n^2)$  vs.  $m \times n$ ). Moreover, every solution of P' is also a solution of dispose(m, n). This can be generalized as follows.

**Proposition 5.** Let  $P = \langle F, O, I, G \rangle$  and  $P' = \langle F, O, I', G \rangle$  be two conformant problems such that  $I \models I'$ . If  $\alpha$  is a solution of P', then  $\alpha$  is also a solution of P.

For a CNF-state  $\varphi$  and a set  $\varphi'$  of clauses in  $\varphi$ , we have that  $\varphi \models \varphi'$ . As such, given that the initial belief state is represented by a CNF-state  $\varphi$ , the above proposition allows one to attempt to solve the problem with a subset of  $\varphi$ . As shown above, this could be significant, especially when there are one-of statements that can be replaced by or statements.

**Definition 8.** Let  $P = \langle F, O, I, G \rangle$  be a planning problem. We define  $P_{\text{one-of}}\langle F, O, I_{\text{one-of}}, G \rangle$  where  $I_{\text{one-of}}$  is obtained from I be replacing every  $\text{one-of}(\ell_1, \ldots, \ell_n)$  in I by  $\text{or}(\ell_1, \ldots, \ell_n)$ .

Proposition 5 indicates that if  $P_{one-of}$  has a solution then it is also a solution of P. However, this cannot be applied arbitrarily as shown in the following example.

**Example 1.** Let  $P = \langle \{f, g, h\}, \{a\}, I, G \rangle$  where  $I = \{ \text{one-of}(f, g, h), \text{one-of}(\neg f, \neg g) \}$ ,  $a : \neg h \rightarrow g$ , and  $G = \{g\}$ . Intuitively, I specifies that  $\neg h$  is true in the initial state as the set of states satisfying I contains only two states  $\{f, \neg g, \neg h\}$  and  $\{\neg f, g, \neg h\}$ . As such, a solution for this problem is a. It is easy to see that replacing the one-of statements by or statements indiscreetly leads to a new problem that has no solution.

The above example shows that one-of relaxation is incomplete. Thus, it is critical to find conditions under which the technique can be used and the completeness of the planner is guaranteed. To this end, we formalize the notion of independence of one-of statements. For a one-of/or statement  $\delta$ , by  $lit(\delta)$  we denote the set of literals occurring in  $\delta$ .

**Definition 9.** A one-of statement  $\delta$  is independent of

- a set of literal S if  $lit(\delta) \cap S = \emptyset$  and  $\overline{lit(\delta)} \cap S = \emptyset$ ;
- an one-of statement (resp. or statement)  $\gamma$  if  $lit(\delta) \cap lit(\gamma) = \emptyset$  and  $\overline{lit(\delta)} \cap lit(\gamma) = \emptyset$ .
- an initial state description I, where  $\delta \in I$ , if  $\delta$  is independent of every element in  $I \setminus {\delta}$ .

Intuitively, the independency of an one-of statement  $\delta$  of I indicates that the literals occurring in  $\delta$  do not appear in any other statements of I. We also need the next definition.

**Definition 10.** Let a be an action and  $\delta$  be an one-of statement  $\delta$ , we say

- a maintains  $\delta$  if one  $|pre(a) \cap lit(\delta)| \leq 1$  and for every state s, if  $s \models \delta$  then  $\Phi(a, s) \models \delta$ .
- a negated  $\delta$  if  $|pre(a) \cap lit(\delta)| \leq 1$  and for every state s, if  $s \models \delta$  then  $\Phi(a, s) \models \overline{\ell}$  for every  $\ell \in \delta$ .

Intuitively, an action maintains an one-of statement  $\delta$  if its execution does not change the overall dependency between the literals in  $\delta$ . In the benchmarks, this type of actions frequently occurs. For instance, the action of *driving* a truck from one location to another location maintains the one-of statement about the locations of the truck; it also maintains the one-of statements about the locations of other trucks, as it does not affect the location of other trucks. To verify the property of maintainability between actions and one-of statements, we make use of the following lemma.

**Lemma 2.** Let  $\delta = \text{one-of}(p_1, \ldots, p_n)$  and a be an action with  $C_a = \{\psi_1 \rightarrow \ell_1, \ldots, \psi_k \rightarrow \ell_k\}$ . Let  $\gamma = \{p_1, \ldots, p_n\}$ and  $L = \{\ell_1, \ldots, \ell_k\}$ . a maintains  $\delta$  if either  $L \cap \gamma = L \cap \overline{\gamma} = \emptyset$ , or the following conditions are satisfied:

 $\circ |L \cap \gamma| \leq 1$ ; and

• if  $\ell_i \in (\overline{L} \cap \gamma)$  then  $\gamma \cap (pre(a) \cup \psi_i) \neq \emptyset$  and  $\psi_i \to \overline{\ell}$ for every  $\ell \in \gamma \cap (pre(a) \cup \psi_i)$ .

The above lemma allows us to check, syntactically, the maintainability of an action and one-of statements. A similar lemma can be developed for checking whether an action negates an one-of statement. We can now state a completeness of the problem  $P_{\text{one-of}}$ .

- **Theorem 2.** Let P be the planning problem  $\langle F, O, I, G \rangle$ . If
- for every one-of statement δ in I, δ is independent of I;
  for every action a in O and one-of statement δ in I, a maintains δ or a negates δ; and
- *G* is a conjunction of literals

then every solution of P is also a solution of  $P_{one-of}$ .

The third condition is important. To see why, consider the statement one-of (f, g) which satisfies  $G = (f \lor g) \land (\neg f \lor \neg g)$ . It is easy to check that or (f, g) does not satisfy G.

We observe that many benchmarks in the literature satisfy the conditions of Theorem 2. For example, the *coins*, *dispose* domains from the recent IPCs satisfy these property.

### **CNF**— A Planner with CNF-States

We now describe CNF, a conformant planner which uses CNF-states as the representation of belief states. The planner is a modification of DNF. Like other heuristic search based planners, CNF implements a best first search algorithm with the following specifics: each node in the search is a pair of a CNF-state  $\varphi$  and the plan p such that  $\varphi = \widehat{\Phi_{CNF}(p,\varphi_I)}$  where  $\varphi_I$  is the initial CNF-state. We omit the search algorithm for space consideration. In what follows, we describe some important considerations, which are different from other planners and contribute to the good performance of CNF.

### **Computing Successor CNF-States**

Given a CNF-state  $\varphi$  and an action a with the set of conditional effects  $C_a$ , computing the successor CNF-state  $\varphi' = \Phi_{CNF}(a, \varphi)$ , by Definition 7, includes the following steps:

- 1. If pre(a) is not true in  $\varphi$  then  $\varphi'$  is undefined.
- 2. If pre(a) is true in  $\varphi$ , then  $\varphi'$  is computed in three steps:

(a)  $enb_a(\varphi)$ (b)  $\Psi = \{Update(\phi, e(a, \phi)) | \phi \in enb_a(\varphi)\}$ (c)  $\varphi' = \times [\Psi]$ 

The algorithms for computing  $Update(\phi, e(a, \phi))$  and  $\times[\Psi]$ are quite straightforward following Def. 4 (for Update), and Def. 3 (for  $\times$ ) so we omit them here. For the simplification of the computation, we implement a preprocessor which combines all effects with the same antecedent into one of the form  $\psi \to \eta$ , where  $\eta$  is a set of literals. The algorithm for computing  $enb_a(\varphi)$  is given in Alg. 1. Observe that it also computes  $e(a, \phi)$  for each  $\phi \in enb_a(\varphi)$ , eliminating some redundant computations. More precisely, the computation in line 7 is done as follows. If  $\phi \models \psi$ then  $Z = \{\phi\}$  and  $e(a, \phi) = e(a, \phi) \cup \eta$ ; If  $\phi \models \neg \psi$ then  $Z = \{\phi\}$  and  $e(a, \phi)$  does not change; otherwise,  $Z = \{\phi_1, \phi_2\}$  where  $\phi_1 = r(\phi \land \psi)$  and  $\phi_2 = r(\phi \land \neg \psi)$ with  $e(a, \phi_1) = e(a, \phi) \cup \eta$  and  $e(a, \phi_2) = e(a, \phi)$ .

Algorithm 1 Computing  $enb_a(\varphi)$ 

1: **Input**: CNF state  $\varphi$ , action *a* 

2: **Output**:  $enb_a(\varphi)$ ,  $e(a, \phi)$  for  $\phi \in enb_a(\varphi)$ 

- 3: Let  $X = \{\varphi\}, e(a, \phi) = \emptyset$  for  $\phi \in enb_a(\varphi)$
- 4: for each effect  $\psi \to \eta$  in  $C_a$  do
- 5: Let  $Y = \emptyset$
- 6: **for** each  $\phi \in X$  **do**
- 7: compute  $Z = \phi + \psi$ , update  $e(a, \phi')$  for  $\phi' \in Z$
- 8:  $Y = Y \cup Z$
- 9: end for
- 10: Set X = Y
- 11: end for
- 12: return X

For computing  $\times [\Psi]$ , we use the following proposition to reduce the amount of computation

**Proposition 6.** Let  $\varphi$  and  $\psi$  be two CNF formulae, c be a clause in  $\varphi$ , and c' be a clause in  $\psi$ . It holds that

- If c = c' then  $r(\varphi \times \psi) = r((\varphi \setminus c) \times (\psi \setminus c') \cup c)$
- If  $c \subset c'$  then  $r(\varphi \times \psi) = r(\varphi \times (\psi \setminus c') \cup c')$

Observe that if  $|\varphi| = n$  ( $\varphi$  has *n* clauses) and  $|\psi| = m$ then the first item says that the size of the cross-product can be reduced by n + m - 1 clauses. The second item indicates that it can be reduced by n - 1 clauses. Our experiments show that this contributes to a significant reduction of search time, ranging between 20% and 90% in the benchmark problems. The main reason is that, without using optimization of Prop. 6, CNF spends most of search time for computing cross-product of CNF-formulae which, if in the same CNFbelief state, usually contains many common clauses since they are originated from the same CNF-formula in the transformation to enabling form. In addition, the *Update* function creates unit clause(s) which can subsume many other clauses in the CNF-states of the same CNF-belief state.

### **Checking Entailment**

Checking whether a CNF-state satisfies a set of literals is needed whenever the planner needs to check for the executability condition of an action and the goal satisfaction of CNF-states. It is also needed during the computation of the enabling CNF-belief states. In general, checking whether a literal  $\ell$  (resp. set, clause) is true or false in a CNF-state  $\varphi$  needs to a call to a SAT-solver. Thus, reducing the number of calls to the SAT-solver is important for the performance of CNF. The representation of belief states as CNF-states provides an easy way to check for entailment in several cases. CNF takes advantages of this fact and reduces the number of calls to the SAT-solver by implementing the following tests before a call to the SAT-solver is made: (i) if  $\ell \notin \varphi$  then  $\varphi \not\models \ell$ ; (ii) if  $\varphi$  contains a clause c' that subsumes a clause c then  $\varphi \models c$ . With this simplifications, the running time portion of the SAT-solver is manageable for most cases. In our experiment, we observe that it is always less than 3%.

# Heuristic

Since CNF is a modification of DNF, it uses the number of satisfied subgoals to guide its search. However, it does not consider the equivalence of the cardinality heuristic. The reasons for this decision are twofold. By using the number of satisfied subgoals, we can evaluate the impact of the belief state representation (using CNF-states) when we compare the performance of CNF and DNF. CNF does not use the cardinality heuristic because of the less the clauses in a CNF-state, the more uncertainty it contains, in general.

# **Preprocessing Input**

CNF is developed by modifying the source code of DNF and CPA. As a result, CNF accepts the same input as DNF and incurs the overhead generated by the preprocessing phase as does DNF. However, CNF does not make use of one-of combination or goal-splitting techniques.

# one-of Relaxation

CNF makes use of the one-of relaxation technique described in the previous section.

# **Experimental Evaluation**

We evaluate CNF against the following conformant planners: DNF, CPA, CFF, POND, and t0 using conformant planning benchmarks found in the literature. To the best of our knowledge, these planners currently yield the best performance in these domains. We obtained DNF, CPA, and t0, from their respective authors. CFF was downloaded from its author website. We use POND version 1.1.1 with default setting.

All the experiments have been carried out using a set of three dedicated Linux workstations of the same configuration: Intel Core 2 Dual 9400 2.66GHz 4GB RAM. The timeout limit is set to 2 hours. The results of our experiments are divided into different tables. In Tables 1, 3, and 4, each column contains the performance of a planner. We report the time and plan length for each planner. 'OM', 'TO', and 'F' denotes out of memory, time out, or abnormal termination of the planner. Due to space limitation, we only report a few large instances of each benchmark and do not report the number of states that are generated and explored by each planner. This number is included in the full version of this paper. In the following, we discuss each table and evaluate the strength and weakness of CNF against other planners.

# **Benchmarks From The Literature**

Table 1 contains the results obtained from experiments on the domains *block*, *raokeys* (raok-n), and *uts-cycle* (uts-c-n) from IPC-2008. *sortnet* is from IPC-2006. *bomb* and *grip*-

*per* are from the author of CFF. The remaining problems including *corner-cube* (cc-n-m), *look-and-grab* (lng-n-m-k), and *sort-number* (sortnum-n) are from the package of t0.

The problems in Table 1 share the common property that they do not contain an one-of statement which satisfies the relaxation condition. Therefore, CNF can not take the advantage of relaxation technique for these problems. The results show that CNF and DNF are the best on the bomb, cornercube, grip, look-and-grab domains and the performance of these two planners are almost the same on these domains. However, CNF scales up better on corner-cube while DNF is better on look-and-grab. The execution time and length of plans found by CNF and DNF are the same for bomb and gripper. For sort-number, t0 is the best on most instances but only CNF is able to deal with the largest instance. We observe that CNF spends more time in transforming CNFbelief states to CNF-states in this problem while the size of the representation of DNF increases very fast. POND outperforms the others on *block* and *sortnet* but its performance is pretty poor on the other domains. CFF is very good at gripper, but it does not scale up well as CNF or DNF does on the larger instances. t0 is the best on raokeys. This planner also performs very well on many small problem instances.

Observe that although DNF outperforms all the other planners on small instances of the *corner-cube* domain but the time increases by a faster rate comparing to CNF. This depends on the number of states generated and explored by DNF and CNF: 24969/5717 v.s. 18321/6362 for *corner-cube-40-20* and 300287/62027 v.s. 115330/39283 for *corner-cube-119-59*, respectively.

On the other hand, CNF does not do so good in the *uts-cycle* domain due to the explosion of the CNF-state size as shown in Table 5. The overhead of the function  $\times$  of converting a CNF-belief state to the equivalent CNF-state is another reason. For example, for the instance uts-cycle-3, the real search time (excluding the input theory translation time) is 3.325 seconds. Whereas, the execution time spent on the function  $\times$  is 3.237, i.e. most of the search time. We present the overhead incurred by CNF for some domains in Table 2. In this table,  $t_trans$ ,  $t_search$ , and  $t_cONF$  are the time used by the preprocessing phase (which DNF does also use), the time for searching for a solution, and the time incurs by the transformation of CNF-belief states to CNF-states.

# **Challenging Problems**

Table 3 contains the experimental results with domains that are challenging for current state-of-the-art planners. Among these, *coins* and *dispose* (ds-m-n) are from the IPC-2006 and IPC-2008. The others are from the challenging domains proposed in (Palacios and Geffner 2007) including *1-dispose* (1d-n-m) and *push* (push-n-m). Indeed, only few planners can solve these problems and none can scale up when the number of objects/constants become large. For coins-21 and larger instances, the main difficulty, which makes these problems challenging, is the huge size of the belief states encountered during the search or the huge size of the initial belief state.

The domains in Table 3 fall in the class of problems where

Problem	CNF	DNF	CPA	Т0	CFF	POND
block-1	0.56/7	0.67/7	0.68/4	0.08/5	0.02/6	<b>0.01</b> /6
block-2	0.83/18	0.68/38	0.76/14	0.2/23	TO	<b>0.06</b> /34
block-3	TO	215/331	OM	48/80	TO	<b>3.9</b> /80
bomb-20-10	0.74/30	0.77/30	3.6/24	F	<b>0.05</b> /30	OM
bomb-50-10	1.28/90	1.28/90	21/58	F	<b>1.16</b> /90	OM
bomb-100-10	<b>2.7</b> /190	<b>2.7</b> /190	110/110	F	34/190	OM
bomb-100-20	5.3/180	<b>5.2</b> /180	244/118	F	28/180	OM
cc-20-10	2.37/138	<b>2.07</b> /217	F	2.1/258	2.5k/332	TO
cc-40-20	13.4/343	<b>7.75</b> /535	F	87/918	TO	TO
cc-64-32	49.5/1106	<b>21.2</b> /872	F	F	TO	TO
cc-99-49	90.6/1542	<b>89.2</b> /1553	F	F	TO	TO
cc-119-59	<b>180</b> /2230	229/3522	F	F	TO	TO
gripper-30	3.3/144	3.7/144	17/72	11/118	<b>0.5</b> /174	ТО
gripper-50	8.6/234	8.7/234	106/106	52/198	<b>3</b> /294	TO
gripper-80	22.2/350	22.5/350	432/166	234/318	<b>18</b> /474	TO
gripper-100	31.9/438	<b>31.2</b> /438	970/206	421/398	>43	ТО
lng-4-1-2	1.21/16	<b>1.06</b> /18	1.17/26	27/12	83/105	OM
lng-4-2-2	1.22/4	1.33/4	1.38/4	31/4	TO	OM
lng-4-2-3	<b>2.8</b> /4	3.13/4	3.91/4	141/4	TO	OM
lng-4-3-3	<b>3.01</b> /4	3.53/4	4.48/4	F	TO	OM
lng-7-2-2	26.4/58	<b>17.9</b> /48	21/28	F	TO	OM
lng-7-3-2	22.9/14	<b>20.8</b> /14	24.7/12	F	TO	OM
raok-2	1.65/27	0.57/26	1.1/32	<b>0.04</b> /21	0.07/34	F
raok-3	2754/919	1.68/153	3.8/152	0.25/66	12/102	F
raok-4	TO	TO	ТО	F	TO	F
sortnet-5	1.2/15	0.94/15	0.94/12	0.26/15	NA	0/12
sortnet-10	21.9/55	1.85/54	3.18/39	OM	NA	0.03/38
sortnet-15	257/119	35.9/118	244/65	F	NA	0.14/65
sortnum-5	1.56/10	2.57/10	OM	1.92/10	2.9/10	<b>0.49</b> /10
sortnum-6	22.8/15	397/15	OM	<b>18.1</b> /15	TO	19.4/15
sortnum-7	539/21	OM	OM	<b>91</b> /21	TO	1077/21
sortnum-8	<b>14277</b> /28	OM	OM	F	TO	ТО
uts-c-3	4.22/3	0.54/3	1.19/3	<b>0.15</b> /3	NA	F
uts-c-4	TO	0.56/6	18.3/6	<b>0.47</b> /7	NA	F
uts-c-5	TO	0.73/10	OM	1.8/10	NA	F

### Table 1: Benchmarks from Literature

the one-of relaxation is applicable. As we can see, CNF outperforms other planners in many challenging domains (*dispose, 1-dispose, and push*) and shows great performance in the *coins* domain. In fact, only CNF was able to solve the instances (-21 to -30) of the *coins* domain. In smaller instances, CNF and DNF are comparable but the high memory usage of DNF results in the "OM" error of DNF in large instances.

### **New Challenging Problems**

In this subsection, we introduce a new set of problems that play to the advantage of CNF. They are variants of the challenging problems of the problems in Table 3. Instead of one-of statements, we use the or statements. To illustrate the modification, let us use the *dispose* problem.

	r	,			J		· · , ~ m
Problem	t_trans	t_search	t_CNF	Problem	t_trans	t_search	t_CNF
block-1	0.53	0.029	0.013	b-50-10	1.01	0.272	0.032
gripper-30	1.19	2.11	0.23	c-20-40	0.75	1.62	0.83
sortnet-5	0.85	0.35	0.15	lng-4-1-2	0.96	0.25	0.11
uts-c-3	0.895	3.325	3.237	sortnum-5	0.613	0.947	0.682

In this problem, there are m different objects  $o_1, \ldots, o_m$ 

 Table 2: Time Distribution: Search vs. Translation vs. Transformation in CNF

Problem	Problem CNF		CPA	Т0	CFF	POND
coins-10	0.6/35	0.62/27	0.9/36	0.04/26	0.12/38	0.5/46
coins-15	coins-15 0.91/81		7/362	7/362 0.12/79		10/124
coins-20	1.15/114	1.41/99	17/105	0.15/107	16/143	105/153
coins-21	1478/7321	OM	OM	F	TO	TO
coins-25	195/1843	OM	OM	F	TO	ТО
coins-29	999/5368	OM	OM	F	TO	ТО
ds-4-3	0.88/93	1.2/185	4.9/92	<b>0.2</b> /122	0.6/73	55/125
ds-4-5	1.15/155	1.7/180	11.3/126	<b>0.8</b> /145	2.5/107	ТО
ds-8-3	27.5/392	37.4/447	778/413	133/761	TO	OM
ds-8-5	32/537	65/878	2152/541	F	TO	OM
ds-10-3	158/531	193/680	4694/648	2388/1360	TO	OM
ds-10-5	175/840	274/1286	TO	F	TO	ТО
ds-10-9	222/1543	787/2178	TO	F	TO	ТО
1d-3-2	<b>0.74</b> /60	0.78/36	0.76/27	0.9/38	TO	5.8/54
1d-3-3	1.04/60	1.86/36	1.87/27	48/36	TO	77/50
1d-3-4	<b>1.67</b> /60	12.5/36	14.3/27	F	TO	OM
1d-6-2	<b>10.8</b> /492	15.4/186	33.7/124	F	TO	OM
1d-6-3	<b>18.1</b> /492	506/186	OM	F	ТО	OM
1d-6-4	<b>30.4</b> /492	OM	OM	F	TO	OM
1d-10-4	<b>667</b> /3806	OM	OM	F	TO	OM
1d-10-5	1240/3806	OM	OM	F	TO	OM
push-4-1	0.94/41	1.17/41	1.61/51	<b>0.16</b> /78	0.28/46	7.3/65
push-4-3	1.06/105	2.1/194	144/847	<b>0.66</b> /164	1.3/48	ТО
push-4-5	<b>1.45</b> /145	5.05/331	OM	2.04/260	126/50	OM
push-8-1	25.8/163	27.1/163	29.6/169	63/464	TO	TO
push-8-2	29.6/396	41.2/903	4254/429	F	TO	ТО
push-8-3	31.9/303	74/1477	OM	F	TO	ТО
push10-6	227/1194	2235/6382	OM	F	TO	ТО
push10-7	259/1221	OM	OM	F	TO	ТО

#### Table 3: Challenging Domains

and *n* different places  $p_1, \ldots, p_n$ . The goal is to find a plan for a robot to pickup the objects and drop them at a place where a trash can is available. Initially, we don't have any information about which object at which place. So, in the initial information there are *m* one-of clauses of following form: one-of $(at(o_i, p_1), \ldots, at(o_i, p_n))$  for  $i = 1, \ldots, m$ . The robot can move between places by the actions up, down, left, and right; pickup objects at different locations by the action  $pickup(o_i, p_j)$ ; hold an object  $holding(o_i)$ .

The new problem, denoted by *or-dispose*, is as follows. In this new problem, each object is associated with a type, i.e., each  $o_i$  is an object type, e.g. pens, books ... The predicate  $at(o_i, p_j)$  now means "there exists an object of type  $o_i$ at place  $p_j$ ." Similarly,  $holding(o_i)$  means "holding some object(s) of type  $o_i$ ." The action  $pickup(o_i, p_j)$  allows the robot to pickup all the objects of type  $o_i$  at place  $p_j$ . Suppose, initially, all we know about the objects is that for each object type  $o_i$ , there is at least one instance of  $o_i$  lying in some places  $p_j$ 's. In effect, we replace the one-of statement one-of $(at(o_i, p_1), \ldots, at(o_i, p_n))$  by the or statement or  $(at(o_i, p_1), \ldots, at(o_i, p_n))$  in the domain specification of *dispose*.

Observe that, when one-of relaxation is applied, the description of the initial state of *dispose* becomes exactly the same as that of *or-dispose*. Hence, the size of the CNF state representing the initial belief state does not change from *dispose* to *or-dispose* if the one-of relaxation is applied. As

Problem	CNF	DNF	CPA	T0	CFF	POND
or-coins-10	0.65/35	OM	OM	0.032/26	0.11/38	0.5/46
or-coins-20	1.1/114	OM	OM	0.15/107	15.9/143	86/121
or-coins-21	1486/7321	OM	OM	F	TO	TO
or-ds-4-3	0.86/93	81/117	OM	<b>0.26</b> /122	0.46/73	63.6/126
or-ds-4-5	1.15/155	OM	OM	1.15/145	1.9/107	OM
or-ds-8-3	27.85/392	OM	OM	278/761	TO	OM
or-ds-8-5	31.9/537	OM	OM	F	TO	OM
or-ds-10-9	222/1543	OM	OM	F	TO	OM
or-1d-3-2	<b>0.74</b> /60	1.15/32	OM	F	TO	6.17/50
or-1d-3-4	<b>1.67</b> /60	420/32	OM	F	TO	OM
or-1d-6-2	<b>10.8</b> /492	190/150	OM	F	TO	OM
or-1d-6-3	<b>18.1</b> /492	OM	OM	F	TO	OM
or-1d-10-5	1240/3806	OM	OM	F	TO	OM
or-push-4-1	0.94/41	0.83/61	52/49	0.38/78	<b>0.26</b> /46	1.86/65
or-push-4-3	1.06/105	115/147	OM	11/189	0.8/48	125/120
or-push-8-1	<b>26.6</b> /163	27.2/134	OM	2.8k/367	TO	TO
or-push-8-2	<b>29.8</b> /396	OM	OM	TO	TO	TO
or-push-10-7	<b>259</b> /1221	OM	OM	TO	TO	TO

Table 4: New Challenging Domains

such, CNF will be able to solve the new problems without difficulty.

The new set of problems contains also the *or*-version of other problems such as *or-coins*, *or-dispose*, *or-1-dispose*, and *or-push*.

Table 4 displays the results of all planners on the set of new problems. Due to the extremely large size of disjunctive normal form formulae representing belief states in these problems, a DNF belief state representation based planner, e.g. DNF and CPA, appears to have very poor performance on these problems. In these problems, the one-of combination technique is not applicable and therefore DNF and CPA perform poorly on these problems. On the contrary, due to the capability of maintaining a compact size of CNF formulae representing belief states, CNF outperforms impressively not only DNF belief state representation based planners, but also all other competitive state-of-the-art conformant planners. It is worth mentioning that there is no significant difference between the performance of any other planner on these problems in comparison with that on the original problems.

### Size of CNF-states

We define the size of a CNF-state the total of the size of every non-unit clause in the CNF-state. The experiments reveal that the size of successor CNF-states keep decreasing in most problems. The reason is that updating a CNF-state by a literal  $\ell$  removes all the clauses containing  $\ell$  or  $\overline{\ell}$ . On the other hand, most clauses in  $(\varphi_{\ell} - \ell) \times (\varphi_{\overline{\ell}} - \overline{\ell})$  are usually trivial or subsumed by another clause in the formula. Tables 5 and 6 report the size of the initial CNF-state and the average size of the generated CNF-states for several instances. Table 6 also shows the effectiveness of the one-of relaxation by additionally reporting those sizes for some problems in Table 3 when the one-of relaxation is not used.

### **Conclusion and Future Work**

In this paper, we introduce a new conformant planner, CNF, which uses a special type of CNF formulae to represent belief states. The overall performance of CNF on several benchmark problems shows that it is competitive with

Problem	Init./Avg.	Problem	Init./Avg.	Problem	Init./Avg.
block-1	47/39	cc-99-49	12/6	sortnet-10	0/43
block-2	105/145	cc-119-59	12/6	sortnet-15	0/87
bomb-m-n	0/0	lng-4-3-3	768/57	sortnum-5	0/28
uts-c-3	132/2820	lng-7-3-2	4802/134	sortnum-7	0/102

Table 5: Size of CNF-states: Initial vs. Average

					2
Problem	Relaxation	No Relax.	Probem	Relaxation	No Relax.
coins-20	54/28	402/200	ds-8-5	320/121	20k/4.7k
coins-22	170/45	1.7k/1.3k	ds-10-3	300/99	30k/5.7k
1d-3-3	27/26	243/446	push-4-5	80/22	1280/165
1d-6-2	72/25	2592/511	push-8-3	192/64	12k/203

Table 6: Size of CNF-states: Applying Relax. vs. Not Applying other state-of-the-art conformant planners in many domains. More importantly, the new planner provides a better scalability than all other planners; it can solve larger instances of many problems that were challenging to previous conformant planners. We propose a new technique, called one-of relaxation, aimed at reducing the size of the initial belief state. We also propose a new set of benchmarks that provides new challenges to conformant planners.

Although CNF displays good performance, it does have some weaknesses which need to be addressed. First of all, the overhead incurred in the computation of the  $\times$  function is sometimes larger than the actual time spent for searching for a solution. Furthermore, an improved heuristic is also desirable. Finally, the performance of CNF and DNF suggests that a combination of the two techniques into one planner might yield interesting result.

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