Loop Formulas for Disjunctive Logic Programs

http://www.cs.utexas.edu/~appsmurf/disjunctive.ps

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Background

- Completion [Clark, 1978] vs. Answer Set Semantics [Gelfond and Lifschitz, 1988].
- Under "tightness", two semantics are equivalent.
- Lin/Zhao [2002] introduced the concept of a loop formula. Answer sets for a nondisjunctive program can be characterized as the models of its completion that satisfy the loop formulas.
- SAT-based answer set solvers: ASSAT, CMODELS.

Our Contribution

- We extend Clark's completion and Lin/Zhao's work on loop formulas to disjunctive logic programs. This explains a puzzling feature of Lin/Zhao's work.
- The concept of a tight program and Fages' theorem are extended to disjunctive programs as well.
- SAT-based answer set solvers can be extended to deal with disjunctive programs.

Completion

We assume that every rule of Π has the form

$$A \leftarrow Body$$

where A is a *clause* (disjunction of distinct atoms).

The *completion* of Π, $Comp(\Pi)$, consists of

$$Body \supset A$$

for every rule $A \leftarrow Body$ in Π, and

$$a \supset \bigvee_{\substack{A \leftarrow Body \,\in\, \Pi \\ a \in A}} \left(Body \wedge \bigwedge_{p \in A \setminus \{a\}} \neg p \right)$$

for each atom a.

Examples

$\Pi_1 :\quad p\ ;\ q$

$Comp(\Pi_1)\ :$

$$p \lor q$$
$$p \supset \neg q$$
$$q \supset \neg p$$

A.S. : $\{p\}, \{q\}$

Models : $\{p\}, \{q\}$

$\Pi_2 :$

$$p\ ;\ q$$
$$p \leftarrow q$$
$$q \leftarrow p$$

$Comp(\Pi_2)\ :$

$$p \lor q$$
$$q \supset p$$
$$p \supset q$$
$$p \supset \neg q \lor q$$
$$q \supset \neg p \lor p$$

A.S. : $\{p, q\}$

Models : $\{p, q\}$

$\Pi_3:$ $p\ ;\ r \leftarrow q$

$q \leftarrow p$

$p \leftarrow \ not\ \ r$

$r \leftarrow r$

$Comp(\Pi_3)\ :$ $q \supset p \vee r$

$p \supset q$

$\neg r \supset p$

$r \supset r$

$p \supset (q \wedge \neg r) \vee \neg r$

$q \supset p$

$r \supset (q \wedge \neg p) \vee r$

A.S. : $\{p, q\}$

Models : $\{p, q\}$, $\{r\}$

Proposition 1 *For any program* Π *and any set* X *of atoms, if* X *is an answer set for* Π*, then* X *is a model of* $Comp(\Pi)$*.*

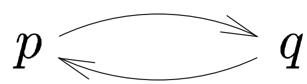
Positive Dependency Graph

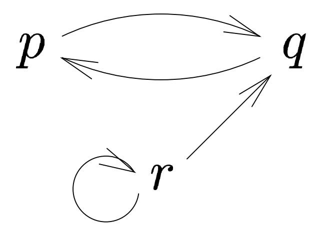
The *positive dependency graph* of Π is the directed graph G such that

- the vertices of G are the atoms occurring in Π, and
- for every rule $A \leftarrow Body$ in Π, G has an edge from each atom in A to each atom in $pa(Body)$.

$pa(Body)$: the set of its "positive atoms". the set of all atoms a such that at least one occurrence of a in $Body$ is not in the scope of negation as failure.

Π_1 : $p\ ;\ q$	Π_2 : $p\ ;\ q$	Π_3 : $p\ ;\ r \leftarrow q$
	$p \leftarrow q$	$q \leftarrow p$
	$q \leftarrow p$	$p \leftarrow$ *not* r
		$r \leftarrow r$





Loops : no loops $\qquad \{p, q\} \qquad \{p, q\}, \{r\}$

Loop Formulas

- We assume that every rule of Π has the form

$$A \leftarrow B, F$$

where A is a clause, B is a list of atoms; every occurrence of each atom in F is in the scope of negation as failure.

- For any loop L of Π, by $R(L)$ we denote the set of formulas

$$B \wedge F \wedge \bigwedge_{p \in A \setminus L} \neg p$$

for all rules $A \leftarrow B, F$ in Π such that $A \cap L \neq \emptyset$ and $B \cap L = \emptyset$.

$$\Pi_2 : \quad \begin{array}{l} p \; ; \; q \\ p \leftarrow q \\ q \leftarrow p \end{array} \qquad\qquad \Pi_3 : \quad \begin{array}{l} p \; ; \; r \leftarrow q \\ q \leftarrow p \\ p \leftarrow \; not \;\; r \\ r \leftarrow r \end{array}$$

$\Pi_2 : \; R(\{p, q\}) \, : \, \{\top\}$

$\Pi_3 : \; R(\{p, q\}) \, : \, \{\neg r\}, \;\; R(\{r\}) \, : \, \{q \wedge \neg p\}$

Loop Formulas—cont'd

$CLF(L)$: the *conjunctive loop formula* for L:

$$CLF(L) = \bigwedge L \supset \bigvee R(L).$$

$CLF(\Pi)$: the set of all conjunctive loop formulas for Π:

$$CLF(\Pi) = \{CLF(L) \ : \ L \textit{ is a loop of } \Pi\}.$$

$\Pi_2 : \ R(\{p,q\}) : \{\top\}$

$CLF(\Pi_2) = \{p \wedge q \supset \top\}$

$\Pi_3 : \ R(\{p,q\}) : \{\neg r\}, \ R(\{r\}) : \{q \wedge \neg p\}$

$CLF(\Pi_3) = \{p \wedge q \supset \neg r, \ r \supset q \wedge \neg p\}$

Theorem 1 *For any program* Π *and any set* X *of atoms,* X *is an answer set for* Π *iff* X *is a model of* $Comp(\Pi) \cup CLF(\Pi)$.

Absolutely Tight Program

A program is *absolutely tight* if it has no loops.

Corollary 1 *For any absolutely tight program* Π *and any set* X *of atoms,* X *is an answer set for* Π *iff* X *satisfies* $Comp(\Pi)$.

$\Pi_1 :\quad p\ ;\ q$

$Comp(\Pi_1) :\quad p \vee q$

$p \supset \neg q$

$q \supset \neg p$

A.S. : $\{p\}, \{q\}$

Models : $\{p\}, \{q\}$