

## BASICS

Sets form the basis of several more advanced forms to be used in our study of semantics. Relations, functions, products and sums are all based on the idea of two sets and an operation that combines the elements of each set in some special way.

## BASIC FACTS

- A collection of objects:  $\{\dots\}$ , where the elements can be any distinct object, no two of which are alike.
- Empty set:  $\emptyset$  or  $\{ \}$
- Elements can share a property:  $\{x \mid P(x)\}$ , where property  $P$  is true of every element  $x$ .
- Can have a set of sets:  $\{x \mid x \text{ is a set}\}$ .

## OPERATIONS

- Element-of:  $x \in S$  is true if  $x$  is a member of set  $S$ .
- Subset:  $S \subseteq T$  is true if  $\forall x \in S, x \in T$
- Proper subset:  $S \subset T$  is true if  $\forall x \in S, x \in T$  and  $\exists x \in T, x \notin S$ .
- Union:  $S \cup T = \{x \mid x \in S \text{ or } x \in T\}$ .
- Intersection:  $S \cap T = \{x \mid x \in S \text{ and } x \in T\}$ .
- Difference:  $S - T = \{x \mid x \in S \text{ and } x \notin T\}$ .
- Grand union:  $\bigcup S = \{x \mid \exists T \in S \text{ and } x \in T\}$ .
- Grand intersection:  $\bigcap S = \{x \mid \forall T \in S \text{ and } x \in T\}$