

BASICS

A relation ρ is a set of primitive pairs $[x, y]$, where x and y are taken from the same or different sets.

Another way to write it is as an infix operator: $x \rho y$, where $[x, y] \in \rho$

ρ is an identity relation if $\forall x, y. x \rho y \Rightarrow x = y$

Given a set S , the identity relation on S is $I_S \triangleq \{[x, x] \mid x \in S\}$

The domain of ρ : $\text{dom } \rho \triangleq \{x \mid \exists y. x \rho y\}$

The range of ρ : $\text{ran } \rho \triangleq \{x \mid \exists y. y \rho x\}$

composition of two relations: $\rho' \cdot \rho \triangleq \{[x, z] \mid \exists y. x \rho y \text{ and } y \rho' z\}$

PROPERTIES

- $(\rho_3 \cdot \rho_2) \cdot \rho_1 = \rho_3 \cdot (\rho_2 \cdot \rho_1)$
- $\rho \cdot I_S \subseteq \rho \supseteq I_T \cdot \rho$
- $\text{dom } I_S = S = \text{ran } I_S$
- $I_T \cdot I_S = I_{T \cap S}$
- $\rho \cdot \{\} = \{\} = \{\} \cdot \rho$
- $\text{dom } \rho = \{\} \Rightarrow \rho = \{\}$
- $I_{\{\}} = \{\}$

THE UNIVERSAL RELATION

If all possible pairings are present a relation, then this is called the *universal relation*. Another term for this is the Cartesian product, written $S \times T$. Any relation $[x, y]$ is a subset of this:

$$[x, y]. x \in S, y \in T \subseteq S \times T.$$

This can be generalized to the n-ary case:

$$[x_1, \dots, x_n]. x_1 \in S_1, \dots, x_n \in S_n \subseteq S_1 \times \dots \times S_n$$

This can also be written as a *product*: $\prod_{i=1}^n S_i = S_1 \times \dots \times S_n$