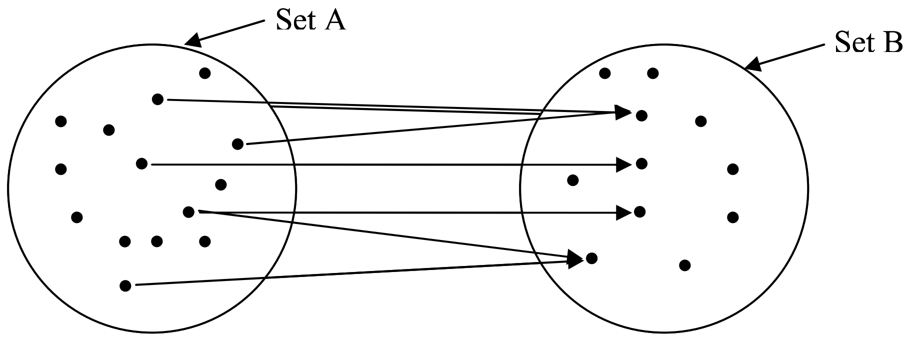
EXAMPLES OF RELATIONS

Relations can be completely arbitrary in which objects are chosen to be related – some elements in either set can be omitted from the relation, and objects can participate in more than one pairing, either way:



We could write this relation as $A\,\rho\,B$. The Cartesian product, written as $A \times B$ connects every object in one set to every object in the other set. Clearly, then, $A\,\rho\,B \subseteq A \times B$.

A simple example is the relation 'greater-than' between natural numbers (i.e. the set $\{0,1,2,3,\ldots\}$). This could be written as: $\{[x,y] \mid \forall x, y \in \mathbb{N}, x > y\}$ or, equivalently as $\forall [x,y] \in \mathbb{N} \times \mathbb{N}, x \text{ greaterThan } y \Leftrightarrow x > y$. ($\mathbb{N}$ is a common way to write the set of all natural numbers.) The relation set starts out: $\{[1,0],[2,1],[2,0],[3,2],[3,1],[3,0],\ldots\}$. Notice that the pair is ordered: e.g. $[0,1]$ is not in the relation.

A more complex example is the relation subset on a set of sets *A*: $\forall S,T \in A, S \subseteq T \Leftrightarrow \forall x, x \in S \Rightarrow x \in T$. If *A* is $\{\{a,b,c\},\{a,b\},\{a\}\}$ then *A* is such a set, because $\{a\} \subseteq \{a,b\} \subseteq \{a,b,c\}$. Notice, for instance, it is not the case that $\{a,b\} \subseteq \{a\}$ since it does not satisfy the condition on the membership of each set. On the other hand, the set $\{\{a\},\{b,c\},\{d\}\}$ does not follow the relation since no set is a subset of any other.