BASICS

This is not a course in formal logic, but there are several essential forms that we need to express logical relationships in the three semantic methods we shall study. Logic is about statements, or assertions, that can be either true or false (but nothing else), and their possible relationships. If *A* and *B* are meta-variables standing for any statement, there are these possible useful forms that can be used to express relationships between them:

- $A \wedge B$ (sometimes written as $A \& B$) is logical conjunction, which is true when both *A* and *B* are true, and false otherwise.
- $A \vee B$ (sometimes written as $A \text{ or } B$) is logical disjunction, which is true when either *A* or *B* is true (or both), but false otherwise.
- $A \Rightarrow B$ is logical implication, which is false if *A* is true and *B* is false, and true otherwise. It is often read as "if *A* then *B*" because it is true when *A* and *B* have the same value, true or false. However, it is also true ("trivially" true) when *A* is false and *B* is true, so sometimes we need:
- $A \Leftrightarrow B$, which is the biconditional or equivalence form. It is true only when *A* and *B* have the same value, true or false, and false otherwise. It is equivalent to $(A \Rightarrow B) \wedge (B \Rightarrow A)$.
- ~*A* (sometimes written not *A*, or $\neg A$) is negation. It is true when *A* is false and vice versa.

VARIABLES

Statements about variables (or placeholders), which can range over a set of objects (constants) are also possible. E.g. $(x \leq 3) \wedge (y \leq 7)$, which is true when *x* and *y* take on values that are less than 3 and less than 7 respectively, and false otherwise. 3 and 7 are members of the set (of integers) that *x* and *y* get their possible values from.

PREDICATES

Relationships between variables and/or constants can be expressed as *P(x,y)*. *P* is a predicate, relation or property. *P(x,y)* is true or false (i.e. it becomes a statement) when *x* and *y* are substituted by suitable objects. Sometimes, we need to express relationships involving some or all of the members of a set. The quantifiers do this:

- $\forall x.P(x)$ is the universal quantifier form which is true for all possible substitutions for *x*.
- $\exists x.P(x)$ is the existential quantifier form which is true for at least one possible substitution for x.
- $\forall x, y, \ldots z.P(x, y, \ldots z)$ is an abbreviation for $\forall x \forall y \ldots \forall z.P(x, y, \ldots z)$
- $\exists x, y, \ldots z.P(x, y, \ldots z)$ is an abbreviation for $\exists x \exists y \ldots \exists z.P(x, y, \ldots z)$

INFERENCE

Many formal systems exist for making inferences from one logical truth to another. They are often called proof systems. Each such system comprises a set of rules for forming the steps of a proof. These are the inference rules of the system. There are many rules for each system. The main rule we shall is "modus ponens", which has the following schematic form:

$$\frac{A \quad A \Rightarrow B}{B}$$

Above the line are the two premises "*A* is true" and "*A* implies *B*". If these are assume, then the conclusion, "*B is true*" below the line must also be the case. It actually expresses in a rule form the axiom of logic (i.e. a statement that is true universally, or a tautology) $(A \wedge (A \Rightarrow B)) \Rightarrow B$, which is true for all possible values for *A* and *B*. There are many other rules of inference because there are many possible axioms; some of these we shall use later.