LOGIC EXAMPLES

Complex statements can be constructed by expanding a meta variable in arbitrary ways. Thus: $A \wedge B \rightarrow A \wedge B \vee C$. Precedence of operators is: $\sim, \wedge, \vee, \Rightarrow, \Leftrightarrow$ but this can be changed with parentheses. e.g. $A \wedge (B \vee C)$.

Some statements are are true regardless of the assignment of true or false values. These are tautologies or axioms. Some examples are:

(1.1) $$A \vee \sim A$$

(1.2) $$A \vee B \Leftrightarrow B \vee A$$

(1.3) $$(A \vee B) \vee C \Leftrightarrow A \vee (B \vee C)$$

(1.4) $$A \wedge (A \vee B) \Leftrightarrow \mathrm{A}$$

(1.5) $$A \Rightarrow B \Leftrightarrow \sim A \vee B$$

(1.6) $$\sim (A \vee B) \Leftrightarrow \sim \mathrm{A} \wedge \sim \mathrm{B}$$

There are infinitely many more. Many are useful in proofs. They can all be proved as tautologies by a truth table method. The basic connectives have the following truth tables:

A	$\wedge$	B
t	t	t
t	f	f
f	f	t
f	f	f

A	$\vee$	B
t	t	t
t	t	f
f	t	t
f	f	f

$\sim$	A
f	t
t	f

A	$\Rightarrow$	B
t	t	t
f	t	t
t	f	f
f	t	f

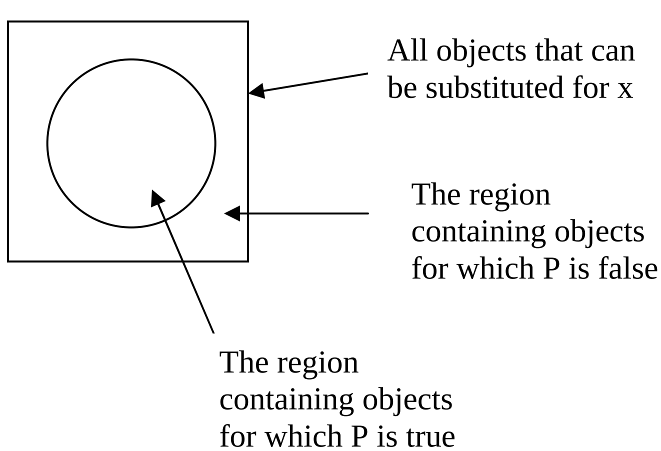
A	$\Leftrightarrow$	B
t	t	t
t	f	f
f	f	t
f	t	f

Each row is a differrent combination of truth assignments to the meta-variables, and the entry under the connective is the resultant truth value of the expression. A tautology like 1.5, can then be proved:

A	$\Rightarrow$	B	$\Leftrightarrow$	$\sim$	A	$\vee$	B
t	t	t	t	f	t	t	t
t	f	f	t	f	t	f	f
f	t	t	t	t	f	t	t
f	t	f	t	t	f	t	f

Note that the column under $\Leftrightarrow$ is all true, which proves the tautology.

There are also tautologies involving the quantifiers. There are two main ones: $\sim \forall x.P(x) \Leftrightarrow \exists x. \sim P(x)$ and $\sim \exists x.P(x) \Leftrightarrow \forall x. \sim P(x)$. These can be 'proved' by appealing to the following diagram:



If it is not true that P is true for all objects, then there must be at least one object outside the circle but inside the square, i.e. for which P is false. Conversely, if it is not true that there is at least one object inside the circle, then the circle must vanish and P is false for all objects in the square.