

Lambda Calculus Practice- Answers

$$\begin{aligned} 1. & ((\lambda x.(x y))(\lambda z.z)) \\ &= [(\lambda z.z)\backslash x](x y) \\ &= ((\lambda z.z)y) \\ &= [y/z]z \\ &= y \end{aligned}$$

$$\begin{aligned} 2. & ((\lambda x.((\lambda y.(x y))x))(\lambda z.w)) \\ &= [(\lambda z.w)\backslash x]((\lambda y.(x y))x) \\ &= (\lambda y.((\lambda z.w)y))(\lambda z.w) \\ &= [(\lambda z.w)\backslash y]((\lambda z.w)y) \\ &= (\lambda z.w)(\lambda z.w) \\ &= [(\lambda z.w)\backslash z]w \\ &= w \end{aligned}$$

$$\begin{aligned} 3. & (((\lambda f.(\lambda g.(\lambda x.((f x)(g x)))))(\lambda m.(\lambda n.(n m))))(\lambda n.z))p) \\ &= ((([\lambda m.(\lambda n.(n m))\backslash f](\lambda g.(\lambda x.((f x)(g x)))))(\lambda n.z))p) \\ &= (((\lambda g.(\lambda x.(((\lambda m.(\lambda n.(n m)))x)(g x)))))(\lambda n.z))p) \\ &= ([(\lambda n.z)\backslash g](\lambda x.(((\lambda m.(\lambda n.(n m)))x)(g x)))p) \\ &= ((\lambda x.(((\lambda m.(\lambda n.(n m)))x)((\lambda n.z)x)))p) \\ &= ((\lambda x.(((\lambda m.(\lambda n.(n m)))x)[x\backslash n]z))p) \\ &= ((\lambda x.([x\backslash m](\lambda n.(n m))z))p) \\ &= ((\lambda x.((\lambda n.(n x))z))p) \\ &= ((\lambda x.[z\backslash n](n x))p) \\ &= ((\lambda x.(z x))p) \\ &= [p\backslash x](z x) \\ &= (z p) \end{aligned}$$

4. this doesn't reduce at all using the β rule e.g.

$$\begin{aligned} & ((\lambda x.(x x))(\lambda x.(x x))) \\ &= ((\lambda x.(x x))(\lambda x.(x x))) \text{ etc.} \end{aligned}$$

$$\begin{aligned} 5. & ((\lambda f.((\lambda g.((f f)g))(\lambda h.(k h))))(\lambda x.(\lambda y.y))) \\ &= [(\lambda x.(\lambda y.y))\backslash f]((\lambda g.((f f)g))(\lambda h.(k h))) \\ &= ((\lambda g.(((\lambda x.(\lambda y.y))(\lambda x.(\lambda y.y)))g))(\lambda h.(k h))) \\ &= [(\lambda h.(k h))\backslash g](((\lambda x.(\lambda y.y))(\lambda x.(\lambda y.y)))g) \\ &= (((\lambda x.(\lambda y.y))(\lambda x.(\lambda y.y)))(\lambda h.(k h))) \\ &= ([(\lambda x.(\lambda y.y))\backslash x](\lambda y.y)(\lambda h.(k h))) \\ &= ((\lambda y.y)(\lambda h.(k h))) \\ &= [(\lambda h.(k h))\backslash y]y \\ &= (\lambda h.(k h)) \end{aligned}$$

6. this does not reduce -it grows! E.g.

$$(\lambda g.((\lambda f.((\lambda x.(f (x x)))(\lambda x.(f (x x)))) g))$$

$= (\lambda g. ((\lambda f. ((\lambda x. (f (x x)) \backslash x] (f (x x)))))) g)$
 $= (\lambda g. ((\lambda f. ((f ((\lambda x. (f (x x)) (\lambda x. (f (x x))))))) g))$
 $= (\lambda g. ((\lambda f. ((f ((\lambda x. (f (x x)) \backslash x] (f (x x)))))) g))$
 $= (\lambda g. ((\lambda f. ((f (f ((\lambda x. (f (x x)) (\lambda x. (f (x x))))))) g)) \text{ etc.}$