NUMBER SYSTEMS

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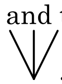
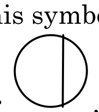
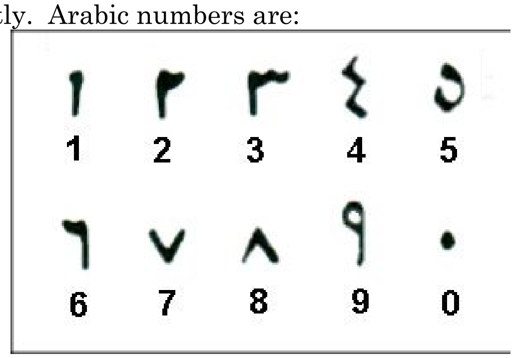
People use number systems that are "convenient." The Babylonians used a sexigesimal system because it was convenient for their purposes. In the author's opinion the reason was that they were concerned about calendar computations and a base of sixty gave them a convenient way to represent the number three hundred sixty. Remnants of this system are still in use today: time - 60 seconds in a minute, 60 minutes in an hour; circular measure – 60 seconds in a minute, 60 minutes in a degree.

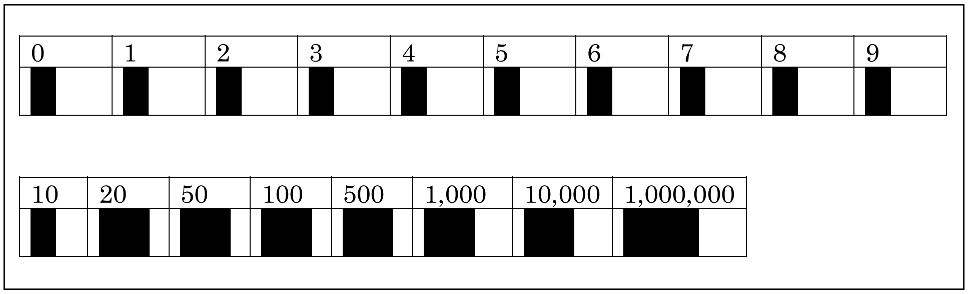
The Babylonians wrote on clay tablets with a stylus. It was convenient for them to use the stylus to represent numbers. By placing the stylus with the point down they could make a mark that represented the number "one" (▼). By placing the stylus sideways they could represent the number "ten" (◄). By combining these symbols in groups they could represent numbers in the sexigesimal system. Later they developed a symbol for "zero."

Roy Underhill, the Woodwright, and chief carpenter at Williamsburg, uses a computer to keep track of items found in archeological digs, e.g. the number of nails at a site. But he uses Roman numerals to distinguish joints in a reconstruction. In a television show, he was asked, "Why?" He replied by holding a very wide chisel and asking the questioner "Have you ever tried to cut a curve with one of these?" The Romans developed their representation of numbers, in the author's opinion, because they were putting their numbers in hard surfaces, e.g. buildings, monuments, etc. They did not calculate using those numbers. To perform calculations, they used an abacus. How did their number system develop? They started by using a single vertical stroke, I, to represent "one." By crossing this, they could represent ten, X. It was still confusing to count the number of strokes before ten. They decided to represent five by using half the symbol for ten, V. By drawing a vertical line through the symbol for ten, they could represent one hundred, . This symbol has been found on some old Roman monuments. This developed into < and then the representation we now show as C. Half of a hundred, was half of this symbol, or . This developed into today's L. One thousand was ten times one-hundred or , which developed into M. Half a thousand, five-hundred, was half the symbol for one-thousand or D.

Other cultures used, or still use, different symbols. We call our system, Arabic numerals. Actually, the system came from India. It was brought to the West by Arab traders. The Arabs write the numbers differently. Arabic numbers are:



The Chinese write with a brush and their numbers reflect this. They use a vertical stroke for one. Interestingly, they cross this to get ten. Their traditional numbers are:



All the systems except the Babylonian are based on ten. The Babylonians used sixty and remnants are in use today. There are sixty seconds in a minute, sixty minutes in an hour. An old Arabic arithmetic book showed how to calculate in the sexigesimal system. In it, seconds were subdivided into "thirds."

The Mayans used a mixed system. The base of a number system is denoted the "radix" of the system. The Babylonians used a radix of sixty and we use a radix of ten. The Mayans used two radices, twenty and eighteen. This should not be considered strange. We do a similar thing with time. There are twenty-four hours in a day, not sixty. Also, the months have different numbers of days. In the Mayan system, the Mayans had a symbol for zero. It was their glyph for a clam shell. They used a positional notation. The first position was units and they counted to nineteen before going to the next place. They use a dot for the number one and a horizontal line for the number five. The number twenty was represented by a dot followed by a clam shell. In the second position they counted only to seventeen, thus a dot followed by two clam shells was three-hundred sixty. In the author's opinion this system developed because the Mayans were interested in calendar computations.

Other people have used other symbols to represent numbers. The Greeks and Hebrews used the letters of the alphabet to represent numbers. In the Dark Ages, a monk named Ulfilas used the twenty-seven letters of the Gothic alphabet to represent numbers. The last letter in the Gothic alphabet was the "thorn" symbol, Þ, used to represent the "th" sound.

The binary number system is used with computers because it is convenient to represent electronic states with two numbers, denoting "on" and "off." This is not convenient for people because a lot of characters are needed to represent small numbers. E.g. Thirteen is written as 1101. It takes four binary characters, called bits, to write a number that is written in two characters in decimal, "13." The number one-hundred is 11100 in binary and one-thousand is 1111101000. For people who work with computer systems and need to deal with binary numbers a more convenient method of representation was developed. The first system was octal, a radix of eight. This allowed three binary bits to be represented as one octal "digit." Later the hexadecimal system was used because computers began using groups of eight adjacent bits, called a byte. A group of four bits, a nibble, can be represented by one hexadecimal "digit."

We write decimal numbers in place notation. Each place has a value. The first place has a value of one and is designated the units position. When nine is reached, adding a one means that we write the number as a one and a zero. The one is in the "tens" position, or radix position. The next position has a value of one hundred and is the radix squared. The number 753 means "seven times one-hundred plus five times ten plus three." In any other base, the position notation means that the first position is the units position, next is the radix and then the radix squared, etc. In binary, the number 101 means "one times four plus zero times two plus one" or five. In octal, the number 36 means "three times eight plus six" or thirty. In hexadecimal the number 215 means "two times two-hundred fifty-six plus one times sixteen plus five" or five-hundred thirty-three.

To convert a binary number to an octal number, divide the binary number into groups of three bits beginning on the right. Each group of three bits is an octal digit.

Examples:

1. To convert the binary 101110 to octal, divide the binary number into 101 and 110. The binary 101 is five and the binary 110 is six. The octal number is written 56 and represents the number thirty-eight.
2. To convert 11010, divide it into 011 and 010 and get 32. This is the number twenty-six.
3. To convert 1111101000, divide it into 001, 111, 101, 000 and get 1750.

In hexadecimal the number 10 is sixteen. How are the numbers from ten to fifteen represented? We resort to the system of using the alphabet. Ten is A, eleven is B, twelve is C, thirteen is D, fourteen is E, and fifteen is F. This means that B3 is the representation of one-hundred ninety-five. To convert binary numbers to hexadecimal representation, divide the binary number into groups of four bits and write the digits.

Examples:

1. To convert the number one-hundred sixty eight, written in binary as 10101000, divide the binary number into 1010 and 1000, this is A8.
2. To convert 110111 into hexadecimal, divide it into 0011 and 0111, this is 37 or fifty-five.
3. To convert 1111101000 divide it into 0011, 1110, and 1000, this gives 3E8.

As is seen, it is relatively simple to convert a number in place position notation into decimal place position. Multiply the decimal equivalent of the place position times the decimal equivalent of the value in that place, then add the results.

Examples:

1. Binary 101011 is 1x32 + 0x16 + 1x8 + 0x4 + 1x2 + 1 = 43.
2. Hexadecimal D3C is 13x256 + 3x16 + 12 = 3328 + 48 + 12 = 3388.
3. Octal 4752 is 4x512 + 7x64 + 5x8 + 2 = 2048 + 448 + 40 + 2 = 2538.
4. The base twelve number 203 is 2x144 + 0x12 + 3 = 288 + 3 = 291.

To convert a number in decimal place position format into another base it is necessary to repeatedly divide by the new radix, keep track of the remainders, and then write them in the reverse order.

Examples:

1. To convert 6 to binary

Number	Divisor x quotient	Remainder
6	2x3	0
3	2x1	1
1	2x0	1
		Binary = 110

2. To convert 359 to hexadecimal

Number	Divisor x quotient	Remainder
359	16x22	7
22	16x1	6
1	16x0	1
		Hexadecimal = 167

3. To convert 451 to hexadecimal

Number	Divisor x quotient	Remainder
451	16x28	3
28	16x1	12
1	16x0	1
		Hexadecimal = 1C3