Towards Algebraic Foundations of Algebraic Fuzzy Logic Operations: Aiming at the Minimal Number of Requirements

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In many applications, it is important to use expert knowledge. Experts often describe their knowledge in imprecise ("fuzzy") properties like "small". These terms are imprecise because for a specific size, an expert may be not absolutely confident whether this size is small or not.
To describe such properties, fuzzy logic was invented in which each statement is characterized by a degree of confidence. Usually, this degree is taken from the interval [0,1], where 0 means absolutely false and 1 means absolutely true.

Once we know the degrees d(A) and d(B) of expert confidence in statements A and B, we need to estimate the expert's degree of confidence in composite statements like A&B, A\/B, and "not A".
The functions providing such estimates are called fuzzy logic operations: and-operations (a.k.a. t-norms), or-operations (a.k.a. t-conorms), negation operations, etc. These operations satisfy natural properties: e.g., since A&B means the same as B&A, and A&(B&C) means the same as (A&B)&C, the and-operation must be commutative and associative. There exist a complete descriptions of all the operations that satisfy such properties.

In principle, we can have very complex fuzzy logic operations. In practice, however, mostly simple algebraic operations are used: linear, quadratic, fractional-linear, etc. In this talk, we show that when we classify such algebraic fuzzy logic operations, we do not need to use all the usual properties of these operations. For example, to classify all quadratic t-norms and t-conorms, we do not need associativity; to classify all quadratic negation operations, we do not need monotonicity, etc.