A Brief Look of an Asymptotic Fixed Point Theorem for Nonlinear Contractions

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Let \((M,d)\) be a complete metric space and \(T\) be a self-mapping of \(M\). W.A. Kirk proved a fixed point theorem for continuous asymptotic contraction \(T\). Y.Z. Chen extended Kirk’s theorem by assuming weaker assumptions on \(T\). Also Chen introduced some other condition to replace the assumption on the boundedness of the orbit. However, we found that by assuming \(\{T^n x\}\) is not Cauchy, this condition does not guarantee the existence of subsequences, \(p_k\) and \(q_k\), satisfying \(\lim_k d(T^{p_k} x, T^{q_k} x) = \delta > 0\), which is needed for the proof of the existence of a fixed point. As a remedy, we introduced the weaker condition \(\liminf_n d(T^n x, x) = 0\) for some \(x\) in \(M\), and proved that this condition implies the existence of a fixed point and the convergence of the Picard iterates to this fixed point.