# Inductive Logic Programming Basic Approaches 

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## Outlines

## $\square$ Introduction

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$\square$ A Simple Demo
$\square$ ILP In The Future

## Introduction

> ILP and Machine Learning
> Problem Specification
> An Simple Example
> Different ILP Learners
> ILP Search Space

## Introduction:

## ILP and Machine Learning (1)



Learning concepts inductively and directly from given examples and background knowledge, with first-order logic as the only representation

## ILP and Machine Learning (2)

- is the intersection of inductive Machine Learning and Logic Programming
- inductively and directly learns concepts
- examples, background knowledge and target concepts are all represented in form of first-order logic
- more powerful than propositional learning (because ......)


## ILP Problem Specification

## Given

- training examples $\mathcal{E}, \quad \mathcal{E}=\mathcal{E}^{+} \mathrm{U}_{\mathcal{E}^{-}}$
- background knowledge $\mathcal{B}, \mathcal{B} \mid \neq \mathcal{E}$

Find

- target concepts $\mathscr{H}$, which are complete, $\mathcal{B} \cup \mathcal{H} \neq \mathcal{E}^{+}$, and consistent, $\mathcal{B} \cup \mathscr{H} \cup \mathcal{E}^{-} \mid \neq \Pi$ with respect to $\mathcal{B}$ and $\mathcal{E}$


## Introduction:

## An Simple Example

- Training Examples
$\mathcal{E}^{+}=\{$daughter(mary, ann). daughter(eve, tom). \}
$\mathcal{E}^{-}=\{$daughter(tom, ann). daughter(eve, ann). \}
- Background Knowledge parent(ann, mary). parent(ann, tom). parent(tom, eve). parent(tom,bob). female(ann). female(mary). female(eve).
- Concept learned: daughter $(X, Y) \leftarrow$ female $(X)$, parent $(Y, X)$


## Different ILP Learners

- Single concept learners and multiple concepts learners
- Batch learners and incremental learners
- Non-Interactive learners and interactive learners

Empirical ILP Learners: non-interactive single concept batch learners

## ILP Search Space (1)

## structure

- ILP problem is a search problem
- ILP search space consists of all syntactically legal hypotheses (clauses) constructed from the predicates provided by the background knowledge
- Very big search space


## Introduction:

## ILP Search Space (2) An Example

If target concept : daughter $(X, Y)$, background knowledge : femal(tom),..., parent(tom,bob) then the following hypotheses are in the search space:

$$
\begin{array}{ll}
d(X, Y) & d(X, Y) \leftarrow f(X), p(X, Y) \\
d(X, Y) \leftarrow f(X) & d(X, Y) \leftarrow f(Y), p(X, Y) \\
d(X, Y) \leftarrow f(Y) \\
d(X, Y) \leftarrow-p(X, Y) & d(X, Y) \leftarrow f(X), p(Y, X) \\
d(X, Y) \leftarrow f(Y), p(Y, X) \\
d(X, Y) \leftarrow p(X, X) & d(X, Y) \leftarrow f(X), p(X, X) \\
d(X, Y) \leftarrow f(Y), p(Y, Y) \\
d(X, Y) \leftarrow p(Y, Y) & d(X, Y) \leftarrow f(X), p(X, Z)
\end{array}
$$

## Introduction:

## ILP Search Space (4)

## Generality relations

- More-general-than
- More-specific-than
- No-more-general-than
- No-more-specific-than
(defined based on $\theta$-subsumption)


## ILP Search Space (5)

Operations
Specializations :

- Add literals into the clause body
- Apply substitutions to the clause

Generalizations :

- Remove literals from the clause body
- Apply inverse-substitutions to the clause


## Introduction:

## ILP Search Space (6) Refinement graph <br> $\operatorname{daught}(X, Y) \leftarrow$

| $\operatorname{daught}(X, Y) \leftarrow$ | $\operatorname{daught}(X, Y) \leftarrow$ | $\operatorname{daught}(X, Y) \leftarrow$ | $\operatorname{daught}(X, Y) \leftarrow$ |
| ---: | ---: | ---: | ---: |
| $X=Y$ | female $(X)$ | $\operatorname{parent}(Y, X)$ | $\operatorname{parent}(X, Z)$ |

$\begin{array}{rrr}\operatorname{daught}(X, Y) \leftarrow & \cdots \cdots \cdot & \operatorname{daught}(X, Y) \leftarrow \\ \text { female }(X), & & \text { female }(X), \\ \text { female }(Y) & & \end{array}$

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## Basic Approaches

> Top-down Approaches
> Bottom-up Approaches
> Hybrid Approaches
> Summary

Basic Approaches:

## Top-down Approaches (1)

Overview

- Use specialization
- Keep a refinement graph
- Search from the most general clause down to less general clauses, i.e., top-down,
( potentially adding literals into the clause body)

Basic Approaches:

## Top-down Approaches (2)

Generic Top-down Algorithm (for e.g. FOIL)
E' :=E
H:= Ǿ
repeat
$c:=\top \leftarrow$.
repeat

## specialization

c := refinement(c)
until some criterion is satisfied
$\mathrm{H}:=\mathrm{H} U\{\mathrm{c}\}$
B := B U \{c\}
$E^{\prime}:=E^{\prime}-\{$ positive examples covered by B \}
until some criterion is satisfied

Basic Approaches:

## Bottom-up Approaches (1)

Two main techniques:
> Relative Least General Generalization
$>$ Inverse Resolution
(all use inverse substitution in different ways)

Basic Approaches:

## Bottom-up Approaches (2)

## Relative Least General Generalization (rlgg)

- rlgg is based on least general generalization $\operatorname{lgg}$
- $\operatorname{lgg}($ structure 1, structure2) is defined between any two structures, e.g.,
$\operatorname{lgg}(\operatorname{parent}($ ann,mary $)$, parent(ann,tom)) $=\operatorname{parent}($ ann,$X)$
- rlgg is lgg of two ground atoms Al and A2 with respect to background knowledge $K$,
$r \operatorname{lgg}(A 1, A 2)=\operatorname{lgg}((A 1 \leftarrow K),(A 2 \leftarrow K))$


## Bottom-up Approaches (3) <br> \section*{rlgg based algorithms (for e.g. Golem)}

//assume background knowledge is a set of ground facts
E' := E
H := $\varnothing$
Repeat
Ep := $\quad \varnothing$

## generalization

c := a clause which covers no examples repeat

Ep := randomly pick several pairs of examples from E' - Ep compute rlggs of the pairs using rlgg $(e 1, e 2)=\operatorname{lgg}((e 1 \leftarrow B),(e 2 \leftarrow B))$ compute rlggs of the rlggs obtained above and c $\mathrm{C}:=$ choose the rlgg with the greatest coverage Ep := Ep - \{ those examples covered by c \}
until no more positive examples are covered by c
$\mathrm{H}:=\mathrm{H} \cup\{\mathrm{c}\}$
$B:=B \cup\{c\}$
$E^{\prime}:=E^{\prime}-\{$ positive examples covered by $H$ and $B$ \}
until some criterion is satisfied

Basic Approaches:

## Bottom-up Approaches (4) Inverse Resolution

Resolution:
Given clause c1 and c2, derive resolvent c Inverse resolution:

Given clause cl and resolvent c, derive another clause c2
Two forms:
propositional form
first-order logic form

Basic Approaches:

## Bottom-up Approaches (5) Inverse Resolution

propositional form:
resolution ( find $L$ in $c 1$ and ${ }_{7} L$ in c2 )

$$
\begin{aligned}
& \mathrm{c} 1 \wedge \mathrm{c} 2 \mathrm{~F} \\
& \mathrm{c}=(\mathrm{c} 1-\{L\}) \mathrm{U}(\mathrm{c} 2-\{ \rceil L\})
\end{aligned}
$$

inverse resolution

$$
\mathrm{c} 2=(\mathrm{c}-(\mathrm{c} 1-\{L\})) \cup\{ \rceil L\}
$$

Basic Approaches:

## Bottom-up Approaches (6) <br> Inverse Resolution

propositional form: (an example)


$$
\text { c2: know } V^{7} \text { study }
$$

c1: pass $V_{7}$ know
c: pass $\vee_{7}$ study
c: pass $\vee^{7}$ study

Basic Approaches:

## Bottom-up Approaches (7)

 Inverse Resolutionfirst-order logic form:
resolution

$$
\begin{aligned}
& \text { ( find } L_{1} \text { in } c 1 \text { and } L_{2} \text { in c2 s.t. } L_{1} \Theta_{1}=L_{2} \Theta_{2} \text { ) } \\
& \mathrm{c} \wedge^{\wedge} \mathrm{c} 2 \mid \mathrm{c} \\
& \mathrm{c}=\left(\mathrm{c} 1-\left\{L_{1}\right\}\right) \Theta_{1} \mathrm{U}\left(\mathrm{c} 2-\left\{L_{2}\right\}\right) \Theta_{2}
\end{aligned}
$$

inverse resolution

$$
\begin{aligned}
& \mathrm{c}-\left(\mathrm{c} 1-\left\{L_{1}\right\}\right) \Theta_{1}=\left(\mathrm{c} 2-\left\{L_{2}\right\}\right) \Theta_{2} \\
& L_{2}=L_{1} \Theta_{1} \Theta_{2}^{-1} \\
& \mathrm{c} 2=\left(\mathrm{c}-\left(\mathrm{c} 1-\left\{L_{1}\right\}\right) \Theta_{1}\right) \Theta_{2}^{-1} \cup\left\{7 L_{1} \Theta_{1} \Theta_{2}^{-1}\right\}
\end{aligned}
$$

Basic Approaches:

## Bottom-up Approaches (8) Inverse Resolution

first-order logic form: (an example)
c2: grandchild(bob,X) $\vee \underset{\urcorner}{ }$ father( X, tom $)$
(c1: father(shannon,tom)

$$
\Theta_{1}=\{ \} \quad \Theta_{2}^{-1}=\{\text { shannon } / X\}
$$

(c: grandchild(bob,shannon)

## Bottom-up Approaches (9)

## $\mathrm{E}^{\prime}:=\mathrm{E}$ <br> H:= $\quad$ Ø

while E' $\neq \varnothing$ ' do
e := the next positive example
invs := all the inverse resolutions of $e$ and $B$
$\mathrm{c}:=$ choose the one with the highest accuracy
$\mathrm{H}:=\mathrm{H} U\{\mathrm{c}\}$
$B:=B \cup\{c\}$
E' := E' - \{ positive examples covered by B \}

Basic Approaches:

## Hybrid Approaches (1)

> Use most specific boundary ( Progol )
> Use general and specific boundaries
$>$ Translate into propositional learning problem

## Hybrid Approaches (2)

## Use most specific boundary (Progol)

## E' := E

$\mathrm{H}:=\varnothing$
while E' $\neq \varnothing$ ' do
e := the next positive example
$\perp:=$ the most specific clause from e and B
$\mathrm{c}:=$ top-down search a best clause between $\mathrm{T} \leftarrow$. and $\perp$
$\mathrm{H}:=\mathrm{H} \cup\{\mathrm{c}\}$
$\mathrm{B}:=\mathrm{B} \cup\{\mathrm{c}\}$
E' := E' - \{ positive examples covered by B \}

## Hybrid Approaches (3)

## Use most specific boundary (Progol)

Consider examples one by one, using resolution, sounds bottom-up

Use resolution to construct a lower bound $\perp$ from B and e, instead of a hypothesis directly

Then search from the very top down $\perp$ to find a clause which covers e and does not cover any of the negative examples, sounds

Basic Approaches:

## Hybrid Approaches (4)

 Use most specific boundary (Progol)true.


## Hybrid Approaches (5)

## Maintain a upper bound and a lower bound

E' := E
Bound $_{\text {upper }}=$ true $\leftarrow$.
Bound $_{\text {lower }}=$ fase $\leftarrow$ true .
while $\mathrm{E}^{\prime} \neq \varnothing$ Ø do
e := the next positive example
if $e$ is positive
Bound $_{\text {lower }}=$ generalize $\left(\right.$ Bound $\left._{\text {ower }}\right)$ which covers e but none of $E^{-}$
if $e$ is negative Bound $_{\text {upper }}=$ specialize $\left(\right.$ Bound $\left._{\text {upper }}\right)$ which does not cover e

$$
E^{\prime}:=E^{\prime}-\{e\}
$$

$\mathrm{H}:=\left\{\right.$ those hypotheses in between Bound $_{\text {upper }}$ and Bound $\left._{\text {lower }}\right\}$

Basic Approaches:

## Hybrid Approaches (6)

## Maintain a upper bound and a lower bound

>_Positive examples raise the lower bound up
> Negative examples push the upper bound down

Basic Approaches:
Hybrid Approaches (7) Use upper and lower boundaries
true.

false.

Top-down approaches:

## Summary

## perform specialization operations

search the refinement graph top-down in brute-force, unique starting point successor hypotheses generated based only on the syntax of the current hypothesis representation, independent of the coming data generate-and-test fashion the impact of noisy data is minimized batch mode: all examples are considered simultaneously

Bottom-up approaches: perform generalization operations search is guided bottom-up by inverse resolution, multiple starting points hypotheses generated based on analysis of an individual example example-driven fashion more easily misled by noisy data incremental mode: examples are considered one at a time

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## Sample Problem

Target concept:

$$
\text { daughter }(X, Y)
$$

Background Knowledge:
parent(ann,mary). female(ann).
parent(ann,tom). female(mary).
parent(tom,eve). female(eve).
parent(tom,ian).
Examples:
\% Positive examples \% Negative examples daughter(mary,ann). daughter(tom,ann). daughter(eve,tom). daughter(eve,ann).

## A Simple Demo:

## Progol Output (1)

CProgol Version 4.4
[Testing for contradictions]
[No contradictions found]
[Generalising daughter(mary,ann).]
[Most specific clause is]
daughter(A,B) :- parent(B,A), female(A), female(B).

## Progol Output (2)

[Learning daughter/2 from positive examples]
[C:-9993,8,10000,0 daughter(A,B).]
[C:-9994,8,10000,0 daughter(A,B) :- parent(B,A).]
[C:-9994,8,10000,0 daughter(A,B) :- female(A).]
[C:-19996,4,10000,0 daughter(A,B) :- female(B).]
[C:3,8,2,0 daughter(A,B) :- parent(B,A), female(A).]
[C:-19998,4,10000,0 daughter(A,B) :- parent(B,A), female(B).]
[C:-19998,4,10000,0 daughter(A,B) :- female(A), female(B).]
[7 explored search nodes]
$\mathrm{f}=3, \mathrm{p}=8, \mathrm{n}=2, \mathrm{~h}=0$

## A Simple Demo:

## Progol Output (3)

[Result of search is] daughter(A,B) :- parent(B,A), female(A).
[2 redundant clauses retracted] daughter(A,B) :- parent(B,A), female(A).
[Total number of clauses = 1]
[Time taken 0.00s]

## Outlines

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## ILP In The Future

- Novel search methods
- Incorporation of explicit probabilities
- Special-purpose reasoners
- Parallel implementations (PILP)
- Enhanced human interaction
(handle huge data sets in the future)

Thantesout

