Automata (Spring 2009) Qual Exam

Answer ALL questions

Question 1 (10 points + 10 points)

(a) Let $p : U \longrightarrow \{T, F\}$ and $S \subseteq U$. Consider the following two statements:

$$\forall a \in S \; p(a)$$

and

$$\forall a \; ((a \in S) \wedge p(a)).$$

Note that $\forall a \in S \; p(a)$ states that $p(a)$ is true for all a in the set S.

Are the two statements equivalent? Justify your answer.

(b) Let $p : U \longrightarrow \{T, F\}$ and $S \subseteq U$. Consider the following two statements:

$$\neg(\exists a \in S \; p(a))$$

and

$$\forall a \; ((a \in S) \wedge (\neg p(a))).$$

Note that $\exists a \in S \; p(a)$ states that $p(a)$ is true for some a in the set S.

Are the two statements equivalent? Justify your answer.

Question 2 (15 points + 15 points)

(a) Give a context-free language L such that $\overline{L}$ is also context-free, but L is not regular. Justify your answer.

(b) Give a context-free language L such that $\overline{L}$ is not context-free. Justify your answer.

Question 3 (5 points + 15 points + 10 points + 20 points)

Let $L_n = \{xy \mid x, y \in \{0,1\}^n,\ x = y\}$. That is, L_n consists of strings that are of lengths $2n$ satisfying the condition that $w \in L_n$ iff the first and second halves of w are the same. Examples: $011011 \in L_3$, $100100 \in L_3$, $101011 \notin L_3$, $100110 \notin L_3$, $10111011 \in L_4$, $01000100 \in L_4$, $10110011 \notin L_4$.

(a) Argue that L_n is regular.

(b) Let L be a regular language and m be a positive integer. Let u_i and v_i, $i = 1, 2, \ldots, m$, be two sequences of m strings. Suppose $u_i v_j \in L$ if and only if $i = j$. Show that any NFA for L has at least m states. *Hint: Given an NFA for L, consider the accepting paths for $u_i v_i \in L$, $i = 1, 2, \ldots, m$.*

(c) Show that any NFA for L_n has 2^n states. Hint: use part (b).

(d) Give an $O(n^2)$-state NFA for $\overline{L_n}$.