Automata Theory

1. Let CFL denote the set of all context-free languages and R denote the set of all regular languages.

   (i) Give an example of a language that is context-free but not regular (no proof needed).

   5 pts

   (ii) Prove or disprove: if $L_1 \in CFL \setminus R$ and $L_2 \in CFL \setminus R$ then $L_1 \cup L_2 \in CFL \setminus R$.

   15 pts

2. For a string $x \in 1\{0,1\}^*$, let $bin(x)$ denote the binary number denoted by x. Consider the following language L:

   $L = \{w \in 1\{0,1\}^* \mid |w| \geq 3,\ bin(w_1 w_2 w_3) \text{ divides } bin(w) \text{ exactly } \}$

   where w_1, w_2 and w_3 denote the first, second and third symbols of w respectively.

   For example,

   $101101 \in L$ as 101101 starts with a 1, $bin(101) = 5$ divides $bin(101101) = 45$ exactly but

   $11111 \notin L$ as $bin(111) = 7$ does not divide $bin(11111) = 31$ exactly.

   Prove or disprove: L is regular.

   40 pts

3. Consider the following language L':

   $L' = \{w \in 1\{0,1\}^* \mid |w| \text{ divides } bin(w) \text{ exactly } \}$.

   For example,

   $1100 \in L'$ as w starts with a 1 and $|1100| = 4$ which divides $bin(1100) = 12$ exactly but

   $11100 \notin L'$ as $|11100| = 5$ which does not divide $bin(11100) = 28$ exactly.

   Prove or disprove: L' is a context-free language.

   40 pts

   Hint: Consider strings of type 10^* in L'.