Ph.D. Qualifying Exam (Spring 2007)
Automata and Formal Languages
Answer ALL questions

Closed Book Examination

Question 1.

(a) (15%)
A language L over Σ is said to be *local* if there exists subsets $A, B \subseteq \Sigma$ and $C \subseteq \Sigma^2$ such that $L = (A\Sigma^* \cap \Sigma^* B) - \Sigma^* C \Sigma^*$. Is a local language always regular? Justify your answer.

(b) (25%)
Given an NFA with no ϵ transition, we define the language of accepting paths as follows: An accepting path consists of a sequence of transitions $(q_0, a_0, q_1)(q_1, a_1, q_2) \ldots (q_i, a_i, q_{i+1}) \ldots (q_k, a_k, q_{k+1})$ such that q_0 is a starting state, q_{k+1} is a final state, and for each $i = 0, 1, \ldots, k$, (q_i, a_i, q_{i+1}) is a valid transition of the NFA going from state q_i to state q_{i+1} processing symbol $a_i \in \Sigma$.

Show that the language of accepting paths of an NFA is local.

Question 2.

Consider $L = \{w \in \{a, b\} \mid |w|_a = |w|_b\}$ where $|w|_a$ denotes the number of occurrences of a in w, and $|w|_b$ denotes the number of occurrences of b in w.

(a) (15%)
Give a deterministic PDA with 1 state and 2 stack symbols to recognize L.

(b) (15%)
Give another deterministic PDA with 2 states and 1 stack symbol to recognize L.

(c) (15%)
From your answer to part (a), derive mechanically a CFG for L. Is your CFG unambiguous or ambiguous? Justify your answer.

(d) (15%)
Is the language $L - \{a^n b^n \mid n \geq 0\}$ context free? Justify your answer.