Department of Computer Science Spring 2007 New Mexico State University

Ph.D. Qualification Exam: Analysis of Algorithms

This is a closed book exam. The total score is 100 points. Please answer all questions.

Let $X = (X_1, X_2, \ldots, X_n)$ be a sequence of n real numbers. A *segment* of X is a consecutive section of numbers in X. For example, $(2,3,4)$ is a segment of $(1,2,3,4,5,6)$, while $(2,4,6)$ is not a segment of $(1,2,3,4,5,6)$. A *maximum segment* X^* of X is defined by a segment of X such that the sum of the real numbers in the segment is maximal, *i.e.*,

$$\sum_{X_k \in X^*} X_k = \max_{1 \le i \le j \le n} \sum_{m=i}^{j} X_m$$

Example: The maximum segment of $(4,-5,3,-3,1,2,-2,2,-2,1,5,-1)$ is $(1,2,-2,2,-2,1,5)$ with a maximum sum of 7.

All questions are concerned with different algorithms to find a maximum segment of a given sequence.

(30 points) 1. Design a $\Theta(n^2)$ algorithm Find-A-Maximum-Segment-Quadratic(X,n) to find a maximum segment of X. Justify the running time.

(30 points) 2. Attempting to obtain an algorithm better than quadratic time in finding a maximum segment, Kottiya, a computer science professor, designed the following algorithm:

(1) Compute

$$S_i = \sum_{j=1}^{i} X_i, \quad i = 1, \ldots, n$$

(2) Let

$$i_{\min} = \operatorname*{argmin}_i S_i$$

and

$$i_{\max} = \operatorname*{argmax}_i S_i$$

(3) Professor Kottiya declares that the segment $X_{i_{\min}+1}, \ldots, X_{i_{\max}}$ must be a maximum segment, if $i_{\min} < i_{\max}$.

(4) If $i_{\min} \ge i_{\max}$, find a maximum segment X^{r*} of the reverse of X^r and declare the reverse of X^{r*} as a maximum segment of X. The reverse of X is defined by

$$X^r = (X_n, X_{n-1}, \ldots, X_1)$$

For example, when

$$X = (-3,-2,1,3,6,-8,2),$$

one can obtain

$$S = (-3,-5,-4,-1,5,-3,-1)$$

where $i_{\min} = 2$ and $i_{\max} = 5$. Thus by Professor Kottiya's algorithm, one gets a maximum segment

$$1,3,6$$

(a) What is the running time of Professor Kottiya's algorithm?

(b) If Professor Kottiya applies for tenure citing this algorithm as his major contribution, will it help or hurt his tenure case? Please justify your answer.

(40 points) 3. Design a $\Theta(n)$ algorithm Find-A-Maximum-Segment-Linear(X, n) to find a maximum segment of X. Justify the running time.

Hint. Let Y be a maximum segment of $X_1, \ldots, X_{i-1}$ with the sum of S_Y. Let F be a suffix of $X_1, \ldots, X_{i-1}$ with the maximum sum of S_F. In other words, F gives the largest sum among all suffixes of the first $i-1$ numbers of X. There are three possibilities to update Y when X_i is being considered:

1. $S_F + X_i > S_Y$.
2. $S_F + X_i > 0$.
3. $S_F + X_i \leq 0$.

You may decide what should be done for the three conditions and then design a linear algorithm to do so.