

Ph.D. Qualifying Exam (Spring 2005)

Automata and Formal Languages

Answer ALL questions

Closed Book Examination

Question 1.

A context-free grammar is called linear if the right hand sides of its productions contain **at most** one nonterminal. A language is a linear CFL if it can be generated by a linear context-free grammar.

An example of a linear context-free grammar is: $S \longrightarrow a \mid aAb$, $A \longrightarrow b \mid bSa$

Consider the following pumping lemma for linear CFL:

For every linear CFL L , there is an integer p such that if s is in L and $|s| \geq p$, then there exist strings u, v, x, y, z such that $s = uvxyz$, where $|vy| > 0$, $|uvyz| \leq p$ and $\forall i \geq 0$, $uv^i xy^i z \in L$.

(a) (5%)

Discuss how the pumping lemma for context-free languages differs from the pumping lemma for linear CFL.

(b) (10%)

Briefly sketch the key ideas for proving the pumping lemma of context-free languages.

(c) (25%)

In proving the pumping lemma for linear CFL, the proof differs from the proof of the pumping lemma for context-free languages in one key aspect. Point out the key difference between the two proofs.

(d) (10%)

Give a context-free grammar for $L = \{a^n b^n c^m d^m \mid n, m \geq 0\}$.

(e) (25%)

Using the pumping lemma for linear CFL, prove that L is not a linear CFL.

Question 2.

(a) (10%)

Sketch a proof of the following:

(★)

If R_1 and R_2 are regular languages, then $R_1 \cap R_2$ is also regular.

(b) (15%)

Consider the following claim about *infinite* intersection of regular languages:

(★★)

If, for all integers $i \geq 1$, R_i is a regular language, then the intersection of all the R_i is regular. That is, $\bigcap_{i \geq 1} R_i$ is regular.

Can we prove (★★) by applying mathematical induction to extend (★)?
Explain your answers.