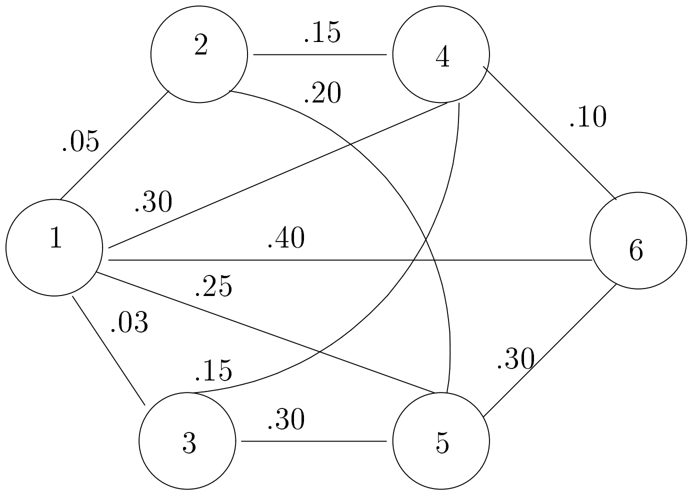
Algorithms

This problem is related to finding paths in communication networks that have the minimum failure probability. A communication network can be represented by a weighted graph $G = (V, E, w)$. Here V, the set of vertices of G, represents the communication points. E, the set of edges, represents the communication links. w is a function from E to $[0, 1]$; for $e \in E$, $w(e)$ is the failure probability for the communication link represented by edge e. We assume that the links are bidirectional and have equal failure probability in each direction. We also assume that link failure events are independent.

Let u, v be two vertices in the network and let $P = e_1, e_2 \dots e_l$ be a communication path from u to v. We say that P fails if any of the links of P fails. Since the edge failure events are independent, the probability of failure of P is given by $1 - (1 - w(e_1))(1 - w(e_2)) \dots (1 - w(e_l))$.

[1.] Consider the communication network below:



Which communication path from node 1 to node 6 has the least probability of failure? Which communication path of length at most 2 from node 1 to node 6 has the lowest probability of failure?

20 pts

[2.] For $k \geq 0$, let $PATHS(u, v, k)$ denote the set of paths from u to v that have at most k edges. Let

$$Best(u, v, k) = min_{P \in PATHS(u,v,k)} \text{ probability of failure of } P.$$

Derive a recurrence relation for $Best(u, v, k)$.

30 pts

[3.] Use the recurrence relation in 2 to devise an efficient algorithm that given a graph G and two vertices u and v outputs a communication path from u to v that has the least failure probability.

(i) Describe clearly the data structures used by your algorithm
(ii) Give pseudocode for your algorithm
(iii) Analyze the running time and the space requirements of your algorithm.

50 pts

Note: Your score on this part will depend on how time and space efficient your algorithm is.