Automata (Fall 2008) Qual Exam

Answer ALL questions

Question 1

Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$, we overload the function name δ to define a function that maps letters in Σ to functions from Q to Q. Specifically, we define $\delta : \Sigma \longrightarrow (Q \longrightarrow Q)$ such that $\delta(a)(q) = q'$ iff $\delta(q, a) = q'$ where $a \in \Sigma$ and $q, q' \in Q$.

We can further extend δ to define $\delta : \Sigma^* \longrightarrow (Q \longrightarrow Q)$ inductively such that $\delta(aw) = \delta(a) \circ \delta(w)$ where $a \in \Sigma$, $w \in \Sigma^*$ and $\circ$ denotes function composition. That is, $\delta(aw)(q) = \delta(a)(\delta(w)(q))$ for $q \in Q$. For the base case of the inductive definition, $\delta(\epsilon)$ is defined to be the identity function such that $\delta(\epsilon)(q) = q$ for $q \in Q$.

With respect to a DFA $M = (Q, \Sigma, \delta, q_0, F)$, we define an equivalence relation $\equiv_M$ on strings in Σ^* such that $w \equiv_M w'$ iff $\delta(w) = \delta(w')$.

With respect to a given language L, we define an equivalence relation $\equiv_L$ on strings in Σ^* such that $w \equiv_L w'$ iff $\forall \alpha, \beta \in \Sigma^*$, $\alpha w \beta \in L \Longleftrightarrow \alpha w' \beta \in L$.

In the following questions, M is assumed to be a DFA.

(a) Show that $\equiv_M$ has finite index; that is, there are finitely many $\equiv_M$ equivalence classes. You are required to give an upper bound on the number of equivalence classes in terms of n, which is the number of states in M.

(b) Show that $w \equiv_M w'$ implies $w \equiv_L w'$ where L is the language of M.

(c) Illustrate with an example to show that $w \equiv_L w'$ does not necessarily imply $w \equiv_M w'$ where L is the language of M.

Question 2

Let Σ be a finite alphabet. For any string $w = a_1a_2 \dots a_n$, where $a_i \in \Sigma$, the reverse of w, written w^R, is the string w in reverse order, $a_n \dots a_2a_1$. For any language L, $L^R = \{w^R \mid w \in L\}$.

(a) If L is regular, is L^R always regular? Justify your answer.

(b) If L is context-free, is L^R always context-free? Justify your answer.

(c) If L is deterministic context-free, is L^R always deterministic context-free? Justify your answer. (Note that a language is deterministic context-free if the language can be recognized by a deterministic pushdown automaton.)