Ph.D. Qualifying Exam (Fall 2007)
Automata and Formal Languages
Answer ALL questions

Closed Book Examination

Question 1.

(a) (10%) Consider a context-free grammar $G = (V, \Sigma, R, S)$ such that each grammar rule is of the form $A \longrightarrow aB$ or $A \longrightarrow a$, where $A, B \in V$ and $a \in \Sigma$. Is $L(G)$ necessarily regular? Justify your answer. If the answer is 'yes', you are required to explain the construction of a finite automaton that recognizes $L(G)$. If the answer is 'no', you are required to provide an example grammar G and argue that $L(G)$ is not regular.

(b) (10%) Consider a context-free grammar $G = (V, \Sigma, R, S)$ such that each grammar rule is of the form $A \longrightarrow Ba$ or $A \longrightarrow a$, where $A, B \in V$ and $a \in \Sigma$. Is $L(G)$ necessarily regular? Justify your answer. If the answer is 'yes', you are required to explain the construction of a finite automaton that recognizes $L(G)$. If the answer is 'no', you are required to provide an example grammar G and argue that $L(G)$ is not regular.

(c) (15%) Consider a context-free grammar $G = (V, \Sigma, R, S)$ such that each grammar rule is of the form $A \longrightarrow aB$, $A \longrightarrow Ba$ or $A \longrightarrow a$, where $A, B \in V$ and $a \in \Sigma$. Is $L(G)$ necessarily regular? Justify your answer. If the answer is 'yes', you are required to explain the construction of a finite automaton that recognizes $L(G)$. If the answer is 'no', you are required to provide an example grammar G and argue that $L(G)$ is not regular.

Question 2.

Let $x \in \Sigma^*$ and $L \subseteq \Sigma^*$. We define $d(x, L) = \{w \mid xw \in L\}$.

(a) Let $L_1 = \{w \in \{a, b\}^* \mid w$ contains twice as many a's as b's $\}$.

1. (15%) Let $L' = \{x \mid d(x, L_1) = L_1\}$. How is L' related to L_1? (No justifications are needed.)
2. (15%) Given that $d(x, L_1) = d(y, L_1)$, how are x and y related? (No justifications are needed.)

(b) (5%) Show that $d(\epsilon, L) = d(\epsilon, d(\epsilon, L))$. Hint: What is $d(\epsilon, L)$?

(c) (15%) Show that $\forall x, y \in \Sigma^*$, $d(xy, L) = d(y, d(x, L))$.

Note that the statement given in part (b) is a special case of the statement in part (c).

(d) (15%) Given L, suppose $Q = \{d(x, L) \mid x \in \Sigma^*\}$ is finite. Define a deterministic finite automaton for L with $|Q|$ states. Hint: Let the DFA be $(Q, \Sigma, \delta, q_0, F)$ where $Q = \{d(x, L) \mid x \in \Sigma^*\}$. What are q_0, F and δ?