

Answers:

1 (a) $L(G)$ is regular. We construct an NFA $(Q, \Sigma, \delta, q_0, \{q_f\})$ for $L(G)$ such that $Q = V \cup \{q_f\}$, $q_0 = S$, $B \in \delta(A, a)$ if $A \rightarrow aB \in R$, and $q_f \in \delta(A, a)$ if $A \rightarrow a \in R$.

1 (b) The reversal of $L(G)$ is captured by a grammar of the form considered in part (a). That is, $L(G)^R$ is regular. As regular languages are closed under reversal, $L(G)$ is also regular. More specifically, we can construct an NFA for $L(G)^R$ according to answer from part (a). By reversing the arrows and switching the starting and accepting states, we obtained an NFA for $L(G)$.

1 (c) No, $L(G)$ may not be regular. Consider the non-regular language $L = \{a^n b^n \mid n \geq 0\}$. We can denote L by the grammar $(\{S, X\}, \Sigma, \{S \rightarrow \epsilon, S \rightarrow aX, X \rightarrow Sb\}, S)$.

2 (a) 1.

$$L' = L_1.$$

2 (a) 2.

$$|x|_a - |y|_a = 2 * (|x|_b - |y|_b)$$

2 (b)

As $\delta(\epsilon, L) = \{w \mid \epsilon w \in L\} = \{w \mid w \in L\} = L$, we have $\delta(\epsilon, \delta(\epsilon, L)) = \delta(\epsilon, L) = L$.

2 (c)

Let $w \in d(xy, L)$. That is, $xyw \in L$. Hence, $yw \in d(x, L)$ which implies that $w \in d(y, d(x, L))$. Thus, $d(xy, L) \subseteq d(y, d(x, L))$.

Let $w \in d(y, d(x, L))$. That is, $yw \in d(x, L)$ which implies that $xyw \in L$; equivalently, $w \in d(xy, L)$. Thus, $d(y, d(x, L)) \subseteq d(xy, L)$.

2 (d)

$q_0 = d(\epsilon, L) = L$, $F = \{d(x, L) \mid \epsilon \in d(x, L), x \in \Sigma^*\}$ and $\delta(d(x, L), a) = d(xa, L)$.