

# Programming Languages

Quals Fall 2006

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## Problem 1: Axiomatic Semantics [35 Points]

Consider the following annotated C code fragment (all variables are integer variables):

```
{ $x \geq 0 \wedge y \geq 0 \wedge x \in \text{Int} \wedge y \in \text{Int}$ }
  a = x;
  b = y;
  pr = 0;
{ $q$ }
  while (b > 0)
  {
    if (b != 2*(b/2))
      pr = pr + a;
    a = a * 2;
    b = b / 2;
  }
{ $r$ }
```

Perform the following tasks:

- describe intuitively what is the result of the computation.
- determine the most appropriate (and simplest) post-condition  $\{r\}$  for the above program.

*Hint: remember that  $'/'$  denotes integer division; consider the cases when  $b$  is even and those where  $b$  is odd.*

- develop an appropriate loop invariant  $\{q\}$ .

*Hint: trace the values of  $a$ ,  $b$ , and  $pr$  at each iteration.*

- Use Hoare's axiomatization to formally prove partial correctness of the above program (using the post-condition and loop invariant determined in the previous two points).

## Problem 2: Language Design and Semantics [65 Points]

Consider a simple imperative language with the following abstract syntax:

```
Program ::= main Block
Block   ::= begin Decl Statement end
Decl    ::= Decl ; Decl
           | var Id
           | procedure Id ( Id ) Block
Statement ::= Statement ; Statement
           | Id = Expression
           | while Expression Statement endwhile
           | Block
           | call Id ( Expression )
Expression ::= Id
           | Nat
           | Expression + Expression
           | Expression > Expression
```

Answer the following questions:

- [55 Points] construct the complete denotational semantics for this language; consider the following guidelines
  - assume that there is only one primitive data type (Nat)
  - do not worry about type checking and error detection
  - it is legal to pass procedure names as arguments to other procedures*
  - your semantics should implement static scoping
- [10 Points] discuss how the passing of procedure names as arguments and their use in call statements could be implemented; make sure to address the problems of variable scoping.