Ph.D. Qualifying Exam (Fall 2005)
Automata and Formal Languages

Answers:

Question 1

(a) We reverse the arrows and reverse the roles of initial and final states to obtain an NFA for L. Next, perform subset construction to turn the NFA into a DFA A. In computing the subset construction, we do not generate all subsets. Compute only the subsets that are reachable.

(b) Let the given DFA be B. As states in A are subsets of states in B, we denote two different states in A by two different subsets of states Q_1 and Q_2 in B respectively. Without loss of generality, let $q \in Q_1$ and $q \notin Q_2$. As B is a reduced DFA, there exists w that causes B to arrive at state q from the starting state of B. Next, with respect to A, the languages L_{Q_1} and L_{Q_2} are different as $w \in L_{Q_1}$ but $w \notin L_{Q_2}$. Therefore, no two states in A are mergeable.

Question 2

Let $L_1 = \{w \in \{a,b,c\}^* \mid |w|_a \neq |w|_b\}$, $L_2 = \{w \in \{a,b,c\}^* \mid |w|_a \neq |w|_c\}$ and $L = L_1 \cup L_2$. It is easy to see that L_1 and L_2 are CFL, and so is L since CFL are closed under union operation.

We claim that L is not deterministic a CFL by showing that its complement is not context-free (since deterministic CFL are closed under complementation).

The complement of L is $\{w \in \{a,b,c\}^* \mid |w|_a = |w|_b = |w|_c\}$. Use the pumping lemma. Pick $s = a^p b^p c^p$. The proof details are the same as that of Example 2.20 in p.117 of the textbook by Sipser.

(Another solution is to consider the language $B = \{a^n b^n c^n \mid n \geq 0\}$ as in Example 2.20 in p.117 of the textbook. It is shown that B is not context-free, hence not deterministic context-free. Let L be the complement of B. One can show that L is context-free. As L's complement is not context-free, L is not a deterministic CFL.)