

Ph.D. Qualifying Exam (Fall 2004)
Automata and Formal Languages
Answer ALL questions

Consider the following pseudo code for processing inputs of strings over $\{0,1\}$.

```
int state = 0; // state is initially set to 0
                // there are 3 possible states: 0,1,2

IntStack S;    // S is a integer stack of 0's and 1's

int i;         // an integer variable holding either 0 or 1

main() {
  initStack(S);
  while ( not end-of-input() ) {

    push( S, getInput() );
    // getInput returns the next input symbol
    // which is either the integer 0 or 1
    //
    // Push the input symbol into the stack S.

  }

  while ( not IsStackEmpty(S) ) {
    i = Pop(S); // the top item in S is popped and saved in i

    if (state == 0 and i == 0)
      state = 2;
    else
      state = (state + i) mod 3;
  }

  if (state == 2)
    return 1; // input is accepted
  else
    return 0; // input is rejected
}
```

Let $L_1 \subseteq \{0,1\}^*$ be the language recognized by the pseudo-code program.

Let $L_2 = \{x0^i y \mid i > |x| \text{ or } i > |y| \text{ where } x, y \in \{0, 1\}^*\}$.

Let $L_3 = \{x0^i y \mid i > |x| \text{ and } i > |y| \text{ where } x, y \in \{0, 1\}^*\}$.

Question 1. (30 points)

Is L_1 regular?

If your answer is yes,
you are required to give a DFA (**deterministic** finite automaton) for L_1 .

If your answer is no, you should provide a proof that L_1 is not regular.

Question 2. (20 points)

Is L_2 regular?

If your answer is yes,
you are required to give a DFA (**deterministic** finite automaton) for L_2 .

If your answer is no, you should provide a proof that L_2 is not regular.

Question 3. (20 points)

Is L_2 context-free?

If your answer is yes, you are required to give a CFG (context-free grammar) for L_2 .

If your answer is no, you should provide a proof that L_2 is not context-free.

Question 4. (30 points)

Is L_3 context-free?

If your answer is yes, you are required to give a CFG (context-free grammar) for L_3 .

If your answer is no, you should provide a proof that L_3 is not context-free.