Automata Theory

1. The following argument shows that all languages are regular:

   Let $L = \{x_1, x_2, x_3 \ldots\}$ be any language. Let $L_i = \{x_i\}$. Then each L_i is regular as it is a finite language. Since regular languages are closed under the union operation, $\bigcup_i L_i$ is regular. Since $L = \bigcup_i L_i$, L is regular.

   What is wrong with this argument?

   15 pts

2. For a language L define:

   $$square(L) = \{xx | x \in L\}.$$

   Prove or disprove: If L is context-free then $square(L)$ is context-free.

   20 pts

3. Consider the following language A:

   $$A = \{x \in \{0,1\}^* | \text{ every prefix of } x \text{ has at least as many 0's as 1's }\}.$$

   Provide a context-free grammar G for A and prove formally that $L(G) = A$.

   40 pts

4. Is the language A in the previous question regular? Provide a proof for your answer.

   25 pts