Ph.D. Qualifying Exam (Fall 1999)
Automata and Formal Languages

Answer ALL questions

Question 1

Let $L = \{w \in \{a,b\}^* \ : \ |w|_a - |w|_b \equiv 1 \pmod 3 \ \}$. Note that w_a denotes the number of occurrences of a in w. Similarly, w_b denotes the number of occurrences of b in w.

Let G be a context-free grammar with the production rules

$$S \longrightarrow AA \mid aAa \mid a$$

and

$$A \longrightarrow SS \mid bSb \mid b$$

where S is the start symbol and $\Sigma = \{a,b\}$.

(a) (15% + 20%) Give a DFA for L. Next, give a regular expression for L by a mechanical conversion from the DFA constructed.

(b) (35%) Using mathematical induction, give a careful proof that $L(G) \subseteq L$.

(In fact, it is also true that $L \subseteq L(G)$. But you are not required to prove this.)

Question 2

Given a language L, we define a relation $\equiv_L$ on Σ^* such that $x \equiv_L y$ iff

$$\forall \alpha, \beta \in \Sigma^*, \alpha x \beta \in L \Longleftrightarrow \alpha y \beta \in L$$

(a) (15%) Prove that $\equiv_L$ is an equivalence relation.

(b) (15%) What are the equivalence classes of $\equiv_L$ for $L = \{a^n b^n \ : \ n \geq 0\}$?