

Historical Projects in Discrete Mathematics and Computer Science

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Instructions for use of projects

A course in discrete or finite mathematics is a relatively recent addition, within the last 30 or 40 years, to the modern American undergraduate curriculum. The course serves to instruct students in precise logical and algorithmic thought, needed for further study of modern mathematics or computer science, and is often required of many secondary education majors, who will teach mathematics in middle or high schools. The roots of discrete mathematics, however, are as old as mathematics itself, with the notion of counting a discrete operation, usually cited as the first mathematical development in ancient cultures [3]. A course in finite mathematics is rarely taught from historical sources, with instruction based on standard textbooks that deliver the modern formulaic versions of many techniques that evolved over centuries. This note outlines the use of historical projects and offers advice for both the instructor and student. All projects along with the Latex source code will be available from the web resource [1].

Teaching from historical sources provides context, motivation and direction to the material, as well as a view of the humanistic aspect of mathematics and a sense of discovery when reading the work of the pioneers. Each historical project is centered around a work or theme of mathematical significance, such as Archimedes' (c. 287–212 B.C.E.) statement about the sums of squares [2] in David Pengelley's project on the summation of numerical powers. The theme of propositional logic and truth tables is treated in a separate project by Jerry Lodder, and draws on sources from Chrysippus (280–206 B.C.E.) [4] to Ludwig Wittgenstein (1889–1951) [5]. A total of fifteen projects will be written:

1. Summation of Numerical Powers
2. Summation of Powers, Bernoulli Numbers, and the Euler-Maclaurin Summation Formula
3. Logic and Truth Tables
4. Boolean Algebra and Discrete Structures
5. Euclid's GCD Algorithm : Recursion vs. Iteration
6. Induction and Recursive Thought
7. A History of Sorting: The Emergence of Quicksort
8. History of Coding and Huffman Codes
9. Networks and Spanning Trees
10. Program Correctness

11. Arthur Cayley and Group Theory
12. Regular Languages and Finite Automata
13. Gödel's Completeness Theorem
14. Peano Arithmetic
15. Gödel's Incompleteness Theorems

The projects are designed to introduce or provide supplementary material for topics in the curriculum, such as combinatorics in a discrete mathematics course, or elementary logic in a computer science course. Each project provides a discussion of the historical exigency of the piece, a few biographical comments about the author, excerpts from the original work, and a sequence of questions to help the student appreciate the source. The main pedagogical idea is to teach and learn certain course topics from the primary historical source, thus recovering motivation for studying the material.

For use in the classroom, allow several weeks per project with one or two projects per course. The time spent working on a project may vary from two to five weeks, depending on the depth and breath in which the material is covered. Some instructors may wish not to assign all parts of a project, while others may wish to add further questions. The source files for the projects may be downloaded from the web resource [1] and edited. Before assignment, the instructor should work through all details of a project, not only for acquaintance with the historical source, but to determine which questions best suit the course material. Pay particular attention to the language of the historical author, and its mathematical content. Be prepared to examine a quote from the source in class and discuss its meaning, either verbally or with present-day formulas. Student solutions may be collected in installments or only once upon completing the project. In either event, monitor student progress on what for them is a lengthy assignment. Each project should count for a significant portion of the course grade (about 20%) and may take the place of an in-class examination. Begin early in the course with a discussion of the relevance of the historical piece, its relation to the course curriculum, and how modern textbook techniques owe their development to problems often posed centuries earlier. Each project offers a "Note to the Instructor" for more detailed information about the level, content and implementation of the project.

The topics covered by the projects are listed above, and range in level from beginning undergraduate courses in discrete mathematics to advanced undergraduate courses in logic, abstract algebra, and computer science. Most are independent of each other, although a few projects build on the same source or present closely related topics via different sources. Most projects state explicitly when relevant material is developed in a separate project that the class may wish to examine. Each historical source is one which either solved an outstanding problem, inaugurated a mathematical technique, or offered a novel point of view on existing material.

Below is a list of instructions to students concerning "How to Work on Your Project." A good student solution should reflect the benefits afforded by close attention to these points. The instructor may wish to photocopy the following instructions for distribution to the entire class.

HOW TO WORK ON YOUR PROJECT: WHAT IS EXPECTED

Here are some words of advice and encouragement for completing the historical project. This is a significant course requirement, in which you will examine a key historical episode in the development of a mathematical topic. The historical presentation is often more verbal than modern textbook formulations, which carries the advantage that no technical knowledge is assumed a priori. On the other hand, you must carefully read the historical author's words and be prepared to experiment with your own calculations to verify or amplify the historical source. Here are some specific instructions.

1. Start today. This is a major assignment, requiring more work than most homework problems from a textbook. Work on the project every day before it is due.
2. Read the entire project to see what is involved. From what time period is the historical piece? What modern topics have evolved from the source?
3. Next read the project very carefully and make a list of any unfamiliar words or concepts. Do you understand the language of the historical author? If not, ask your instructor.
4. Feel free to seek additional historical or mathematical information about the source from either the library or the internet. Always question the validity of internet sources.
5. Begin answering questions in the project. Use complete sentences along with equations, tables, or diagrams to support your work. Some questions will be rather simple, while others require detailed solutions. Identify the difficult questions early and seek help from your instructor if you don't know how to proceed.
6. Seek feedback from your instructor even if you feel that you have answered the questions correctly, since you may have misinterpreted a passage.
7. The final solution to the project should be a written paper in which you address all the issues raised in the project. Use complete sentences along with modern equations to support your claims. Discuss any connections from the historical source with present-day techniques that you may have encountered. How does the historical source differ from the textbook presentation?

After completion of a course using historical projects, students write the following about the benefits of history: “See how the concepts developed and understand the process.” “Learn the roots of what you’ve come to believe in.” “Appropriate question building.” “Helps with English-math conversion.” “It leads me to my own discoveries.”

References

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